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ENGINEERING DESIGN GRAPHICS JOURNAL

SPRING 1981 VOLUME 45 NUMBER 2

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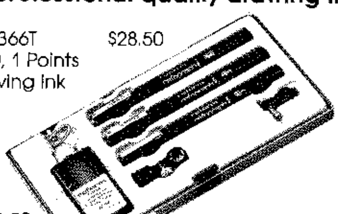

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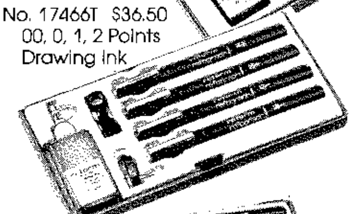

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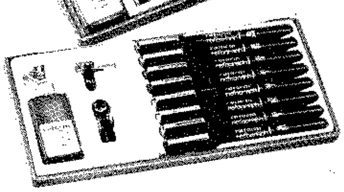

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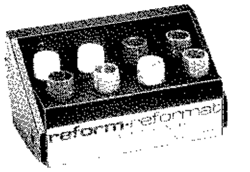

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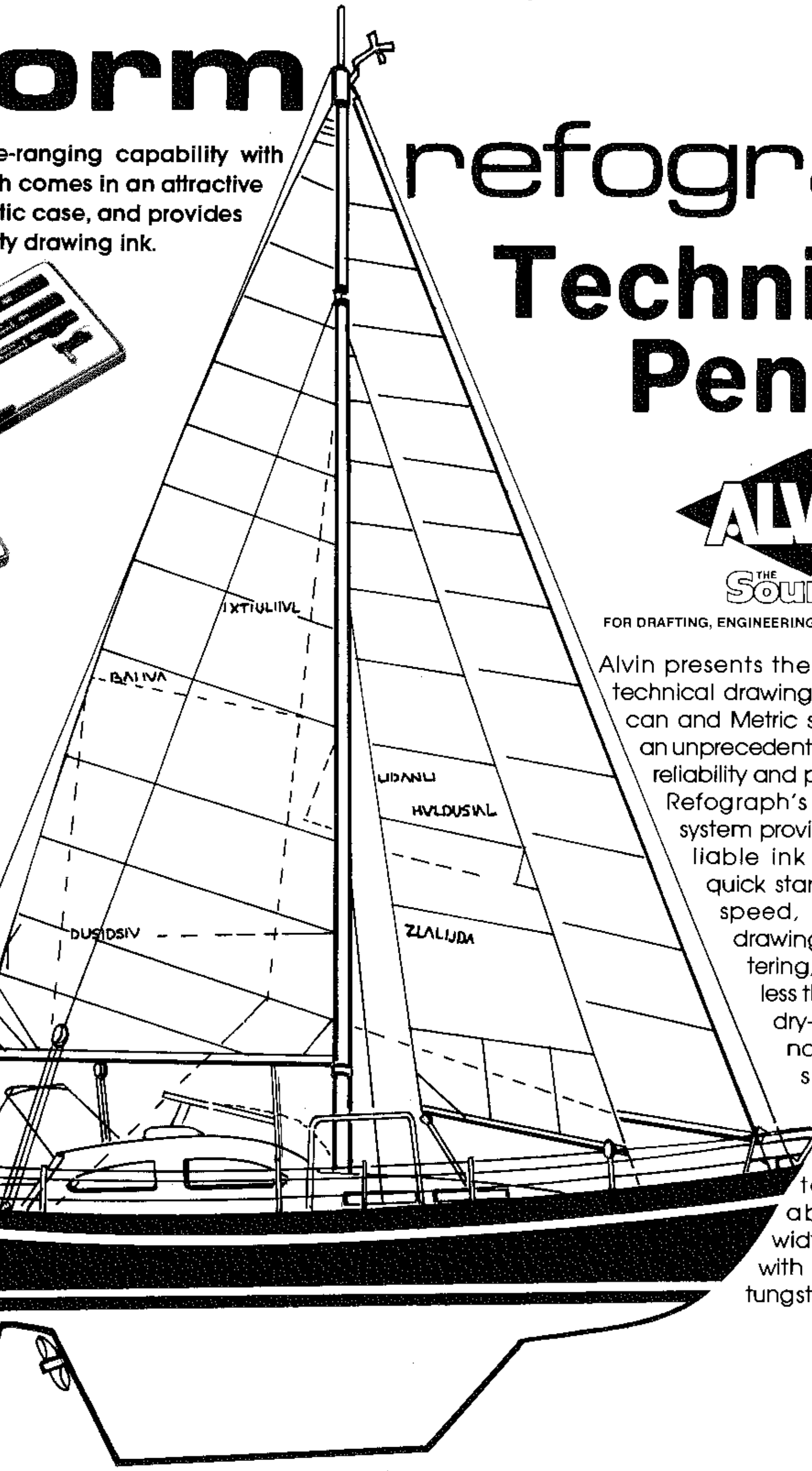

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The views and opinions expressed by the individual authors do not necessarily reflect the editorial policy of the ENGINEERING DESIGN GRAPHICS JOURNAL or of the Engineering Design Graphics Division of the ASEE. The editors make a reasonable effort to verify the technical content of the material published; however, final responsibility for opinions and technical accuracy rests entirely upon the author.

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ENGINEERING DESIGN GRAPHICS JOURNAL OBJECTIVES:

The objectives of the JOURNAL are:
1. To publish articles of interest to teachers and practitioners of Engineering Graphics, Computer Graphics and subjects allied to fundamentals of engineering.
2. To stimulate the preparation of articles and papers on topics of interest to its membership.
3. To encourage teachers of Graphics to innovate on, experiment with, and test apprpriate techniques and topics to further improve quality of and modernize instruction and courses.
4. To encourage research, development, and refinement of theory and applications of engineering graphics for understanding and practice.

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The following deadlines for the submission of articles, announcements, or advertising for the three issues of the JOURNAL are:
Fall . . . . September 15
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The Editor welcomes articles submitted for publication in the JOURNAL. The following is an author style guide for the benefit of anyone wishing to contribute material to the ENGINEERING DESIGN GRAPHICS JOURNAL. In order to save time, expedite the mechanics of publication, and avoid confusion, please adhere to these guidelines.

1. All copy is to be typed, double-spaced, on one side only, on white paper, using a black ribbon.

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3. Two copies of each manuscript are required.

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Accordingly, be sure all lines are sharply drawn, all notations are legible, reproduction black is used throughout, and that everything is clean and unfolded. Do not submit illustrations larger than 198 x 280 mm. If necessary, make 198 x 280 or smaller photocopies for submission.

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REVIEW OF ARTICLES

All articles submitted will be reviewed by several authorities in the field associated with the content of each paper before acceptance. Current newsworty items will not be reviewed in this manner, but will be accepted at the discretion of the editors.

NOTE: The editor, although responsible for copy as it is published, begs forgiveness for all typographical mistakes, mis-spelled words and any goofs in general. Typing is done mostly by non-professional word processors who either are still in high school or are not trained in professional word processing. Thank you for your patience.

CALENDAR

<u>ASEE ANNUAL CONFERENCES</u>

1981 - University of Southern California
Los Angeles, June 22 - 25

1982 - Texas A & M University
College Station, June 21 - 24

1983 -

<u>EDGD MIDYEAR CONFERENCES</u>

1981 - The University of Louisville
Louisville, KY

1982 - California Polytechnic Univ.
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ENGINEERING DESIGN GRAPHICS JOURNAL

THE AMERICAN SOCIETY FOR ENGINEERING EDUCATION

Volume 45 Number 2 Spring 1981

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EDITOR'S PAGE

SHEE!
(with apologies to SHOE)

Many thanks to ERNEST R. WEIDHAAS, (Assistant Dean for Commonwealth Campuses, the Pennsylvania State University) and his son, MARK, for the extraordinary cover photograph of the butterflies. Purists may say that "These are not truly orthographic, as the butterflies do not project horizontally!" The Journal believes that truly orthographic or not, even the black and white photo deserves special recognition. But, oh -- if you could have seen the color photograph! Unfortunately, the Journal budget does not permit a full-color cover. Again, thanks to Mark Weidhaas (for capturing Nature's best) and to Dean Weidhaas for thinking of his friends at the Journal.

This issue is another "theme" issue with articles on basic graphic techniques, theoretical graphics, and descriptive geometry. Of major interest is a translation from the French of Monge's two-view solution of the shortest perpendicular distance between two skew lines. A more modern two-view solution based on the plane method is set forth in the article following the translation. Authors/ Translators are LEIDEL, YING and REYNOLDS (University of Wisconson-Madison). In addition, descriptive beometry buffs will find several solutions for Kelso's "Puzzle Corner" offerings of Fall'80 presented in that section of this issue of the Journal.

For those readers who enjoy theoretical graphics, the Journal is full to overflowing. ROTENBERG (University of Melbourne, Australia), LAND (Miami University of Ohio), BRISSON (Rhode Island School of Design), and REINHARD LEHNERT (with his Gestaldt Graphics) offer a little something for everybody.

DAVE BRISSON has also written our "Guest Editorial" this issue, extolling the virtues of "Descriptive Geometry... as a Liberal Art". The Journal welcomes comments on Prof. Brisson's "soap-box" stand.

On the practical side, readers will find of interest the second article by RAETHE (Queen's University, Kingston, Ontario) concerning graphical plotting of moment diagrams; the long-awaited Oppenheimer Award-winning presentation (Mid-Winter Meeting, Williamsburg, Va., 1980) by LARRY GOSS (Indiana State University-Evansville) on "Consulting in Graphics . . ."; and BILL VANDER WALL'S (North Carolina State University) article on models for visualization.

And, for those interested in the history of graphics, the Journal presents LAGHI'S article (with some translation by the author from German sources) on the beginnings of the present day field of reprographics. Prof. Laghi's name is added to our lengthening list of overseas contributers, and we thank these folks for their continuing interest in our publication.

The "Letters . . ." portion of the Journal has been expanded somewhat this issue, and what is included in those pages will be of great interest to the readers. The editor always gets mail of this nature, but somehow, some of it is misplaced before publication, or something has to be left out for the 16 page divisions required by the printer, so the inclusion of these letters is almost miraculous, and certainly not to be overlooked by the reader.

The Journal Staff welcomes YAAQOV (JACK) ARWAS (The Technion, Haifa, Israel) as an Assistant Editor. Jack is a regular contributer to the Journal and he will be responsible for reviewing and editing contributions from overseas, mainly in the area of computer graphics.

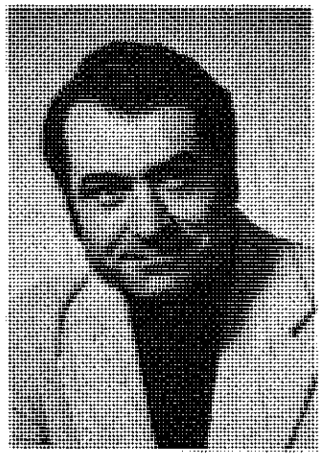
Everyday, since the University's last class of the Spring semester, at least one person in my family has asked, "When are you going to send off the Journal?" Probably some of the readers have asked the same question, but couched in different terms; e.g., "When will I get my Spring Journal???"

Well, here 'tis! Hope you enjoy it!

MARY JASPER

CHAIR'S PAGE

CHAIR'S PAGE
CHAIR'S PAGE
CHAIR'S PAGE
CHAIR'S PAGE



This is my last opportunity to address you as Chair, and I want to thank all of you who have worked for the Division this year - and in previous years. I also want to thank my wife Judy, whom many of you know, for her infinite patience and selfless help with a myriad of tasks at ridiculous hours. She is without equal. To the outgoing officers and committees, I hope you will continue your involvement with the Division in some way; to the incoming, I believe the coming years are going to offer great and perhaps perplexing challenges. Maintaining member contack and flow of information in the face of prohibitive travel costs will provide a tremendous opportunity for development of section activities and the JOURNAL.

We have made an effort this year to bring the work of the Freshman Year Committee to fruition by persuading the NSPE and ABET to recognize, endorse, and we hope ultimately insist upon graphics as a necessary part of an engineer's education. We have realized some measure of success in this effort, and the coming year may also prove fruitful. Otto Tennant, a friend and fellow Iowan will be president of NSPE and understands very well the need for documentation in engineering.

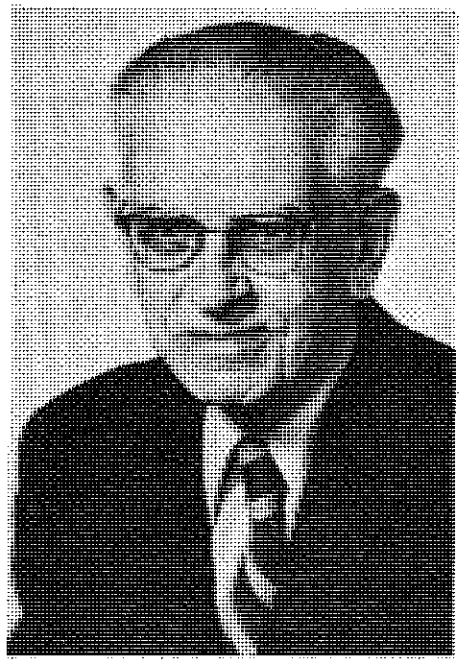
Garland Hilliard has undertaken the immense task of searching out and contacting some recently-lost members and will, as he always does, provide another yet valuable service to the Division.

The Creative Engineering Design Display continues to be an important part of the ASEE Annual Conference, and Bob Foster and his committee have done a yoeman's job with that this year.

The year draws to a close too quickly, and as Claude Westfall once observed about the Chairship of the Division, a year is barely enough time to learn what the job is all about, much less have an influence upon it. That fact speaks very well of our membership and officers. It is probably a rather indirect way of saying that the Division may be able to survive very nicely without its Chair, I suppose.

Thank you again for your support and contributions. Good luck, Jack!

In Memorium . . .



Maurice E. Almfeldt

1903-1981

Maurice E. Almfeldt, Professor Emeritus in Freshman Engineering at Iowa State University, died April 8, 1981 at 77 years of age.

"Al" as we all knew him, was born July 10, 1903 in Lake Park, Minnesota. He received a diploma in 1926 from the Rhode Island School of Design and a Bachelor of Science in Mechanical Engineering from Rhode Island State College in 1932. Al began teaching at Rhode Island State College and was associated with education for the next forty years. He moved from Rhode Island to Syracuse University and then to Iowa State in 1951. From 1951 until his retirement in 1974, Al taught in the Department of Engineering Graphics at Iowa State.

Professor Almfeldt was truly an outstanding teacher. His rapport with students, knowledge of subject matter, careful class preparation and willingness to give extra assistance earned him the highest respect by students and faculty.

In 1966, the engineering students selected Al as the "Engineering Professor of the Year". In 1975 he was awarded the ISU Faculty Citation in recognition of his outstanding teaching and service to the University.

Al's professional contributions included co-authoring several editions of the text Engineering Graphics, together with numerous problem books. He was a long-time active member of the Engineering Design Graphics Division and held several key positions including Division Secretary, member of the Board of Directors and the Executive Committee.

Al was an active member of the Church of God in Ames, Iowa. He was widely known as a licensed amateur ham radio operator and used this hobby to contribute effectively with the Department of Civil Defense both during WWII and in recent years during disasters.

His untarnished moral and ethical principles, his ability as a teacher, and his love of the profession will be missed by students and peers.

We will long remember Al's good-natured, wry and unruffled sense of humor; those thousands of students he guided will always appreciate his genuine contribution to their education. We will miss Al.

Guest Editorial

DESCRIPTIVE GEOMETRY CONSIDERED AS A LIBERAL ART

DAVID W. BRISSON
RHODE ISLAND SCHOOL OF DESIGN
PROVIDENCE, RI

Descriptive geometry is usually considered a service course for engineers. Today there is an increasing tendency to replace basic descriptive geometry with courses in computer graphics. Certainly there are things that may be accomplished by means of the computer that would be difficult if not impossible to accomplish by traditional means. It is not the purpose, however, of this paper to argue the case as to the merits of either discipline as the tool of engineering, but rather to discuss the merits of descriptive geometry as a Liberal Arts subject.

For the past several years the author has taught a course at the Rhode Island School of Design concerned with the visualization of four-dimensional geometry. The students who have participated in this course have been from a great variety of design related disciplines: architecture, industrial design, sculpture, fashion design, painting, etc. The course started as a course of the winter-session program, a special six-week program between two major semesters. This past year it has been introduced into the regular curriculum under the Freshman Foundation Division, and it is planned that during the next academic year it will be placed under the Liberal Arts Division. Much of the course depends on the enrollment, for this is entirely an elective course.

The enrollment in past years in this course has not been large, but sufficient to justify its continuance. In a way, it is somewhat surprising that it has continued at all, for the Rhode Island School of Design is very much an "art" school, and the students are generally suspicious of mathematics, as artists often are. There is a curious mystique associated with the idea of four dimensions because of its cultist associations - which accounts, in part, for its attractiveness to the art student. It conjures up the exotic in a way that three-dimensional descriptive geometry never could; as a matter of fact, the geometry of four dimensions is bizarre in many ways when compared to that of three dimensions. As a simple example, in four dimensions it is possible for two flat planes to have only one point of intersection!

The course itself has not been concerned with descriptive geometry alone, and in fact has not even been described as descriptive geometry as such. Probably if it had been described in such a way, it would not have had as much response. It has been titled "Seeing in Four Dimensions" or "The Perception of Four Dimensions", or in its most recent form, "Hypergraphics", based on the book by that name which was edited by the author in 1979 for the American Association for the Advancement of Science. The course deals with physiological optics, and the use of the hyperstereogram, an analog of the stereogram, and the hyperanaglyph. It involves model-building, and readings from Manning, Ouspensky and Coxeter, and discussions of Zen Buddhism, Taoism and Sufism, digressions into the work of J. B. Rhine and of Jung.

All of this discussion centers on the strict descriptive geometry of four dimensions and more. Nor does it stop there, for it extends into the non-Euclidean geometries and finally into analytic complex geometry. But the basic tools are projective visual geometric methods. It is really quite amazing how fundamental descriptive geometry is to the comprehension, not only of engineering or architectural skills, but of the fundamental philosophical and cultural problems of mankind.

Descriptive geometry, as it may be understood in its most advanced present form, is the tool of visualization. Coupled with computer-graphics, we have available to us the means to run "hyper-circles" around our predecessors in visual thought. For various reasons, which are not altogether clear, our society has tended to emphasize verbal, rather than visual, thought processes in its educational programs. Art courses are often thought of a playtime for wayward students rather than meaningful aspects of a child's education. Mechanical drawing courses are thought of as necessary, but minor, courses for scientifically oriented students and for vocational education, but hardly on a par with the more prestigious algebra. On the higher level of education, geometry is usually given short shrift in relation to the more widely touted analysis course.

As a teacher in art school, it has been the experience of the author to encounter a majority of students who are visually oriented. Most of their lives have been spent in "making pictures." In some ways they are quite sophisticated, even as freshmen, when compared to those who have been less interested in art. They often can draw very accurate projections of what they see in front of them. They can function successfully as human "cameras". It is thus rather surprising to find that about one out of twenty-five such students do not have normal binocular vision, and a similar number are color-blind. This is not a statistical statement, but a matter of practical experience after having encountered several thousand incoming students over the years. As a matter of fact, it has been evident, on occasion, that artists might perhaps become artists as a form of over-compensation for some kind of visual difficulty, particularly since the culture does not encourage people to become artists.

Apart from this unusual skill in reproducing a visual image with precision, it has been the experience of the author that other aspects of the students' visual processes are about as undeveloped on the average as would be expected from the general population, at least, in the beginning. The ability to "visualize" and draw from one's imagination in such a group is relatively rare; that is, one or two from such a group can do it fairly well, contrary to what one would expect.

Inhibited response to the use of color is also rather common to such a group, and in general, they would rather draw than use color. Their knowledge of color theory is nearly universally incorrect, having been usually misinformed by the equally misinformed secondary art teachers, except for the occasional student who had a good physics course. Their knowledge of vision, in general, is weak and after-images often come as a surprise to them; the basic effects of binocular vision is usually a surprise, as well. The author has not yet met a single incoming student who was aware of his blindspots, and often students with rather severe optical problems are not even aware of them.

One of the basic problems faced in teaching such a group of incoming art students is the transferring of the knowledge of the fundamentals of three-dimensional visualization. Although our educational system does a fair job with Euclidean two-dimensional geometry, the learning process usually stops right there. It is indeed rare to find a student who knows what an octahedron is, for example. Most students know the cube, the pyramid, the cylinder, the cone, the sphere and the rectangular solid, and that is it! Most of three-dimensional geometry is a mystery to them. And these are the students who are interested in visual thought very strongly! It is not a lack of ability that is involved in their lack of knowledge, however. One of the first problems that is often given to the students in courses in three-dimensional design is the bisection of a cube, and the results, after several seeks of experimentation and study are often remarkable. It is plain neglect on the part of our educational system which gives rise to the predominant lack of ability to visualize at the baccalaureate level.

Those who are not visually oriented might ask, "What could be the purpose of such training to those other than artists or engineers?" A lab technician or a business executive may never have to recognize an octahedron when they see one; certainly, it must be irrelevant to some that in four dimensions two flat planes may intersect in only one point;

and even engineers really do not need to master the exercises of discriptive geometry because a computer program can do this for them. If an education is understood to be a training program to teach people to do specific narrow jobs (in which they do not initiate change, but only carry out designated and well-described tasks) then the answer is such experiences are irrelevant.

If, on the other hand, an education is understood to be a program which inculcates the basic development of the individual as a whole, and in every direction of his potential, then every basic mental and physical skill should be introduced to that individual. It is a sad fact of our culture, that visual skills are so badly neglected that the average adult has such skills developed to no greater level than a seven year old child. What an enormous loss of potential!

Descriptive geometry is really the rational approach to visual thought. It is so much a necessary part of our technology, that it is taken for granted, and given a minor role with respect to the technical disciplines. It is often though of as a playpen discipline, not as important, or ultimately as difficult as other disciplines. In contest to that, the visualization of the close-packing of hyper-spheres in eight-dimensional space is neither trivial nor easy. Nor is it dull. There is fantastic and incredible fascination in watching the rotation of a simple hypercube in a graphic readout, or in hyperanaglyphic model form.

The practical role aside, there is the sheer mental and emotional development of the individual with which we must be concerned. Being such a basic mental tool, not for just solving technical engineering problems, but in visually comprehending the world, descriptive geometry is, by its fundamental nature, an essential ingredient in a basic education.

The course, described in preceding paragraphs, is an example of how such a course may be implemented. It is probably best that it never was introduced as a course in descriptive geometry, and it is very encouraging that the school has recognized the value of the course to the point that it has been included as a Liberal Arts subject.

It is hoped that success of this one course and the comments concerning this course will encourage others to (1) develop their own strength in the higher geometries and (2) strive for the acceptance of such studies into the basic structure of the educational system.

LOOKING FOR SOMEONE??

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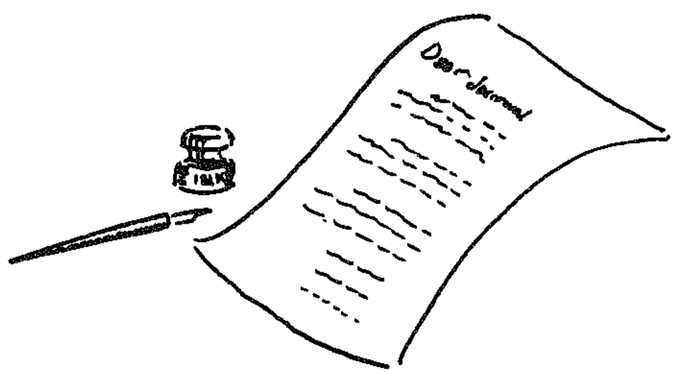
Letters to the Editor



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SEPT. 23, 1980

Professor Mary A. Jasper
The Editor
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Dear Professor Jasper,

I am writing in response to Pat Kelso's open letter (EDGJ, Winter 1980, Page 62) and his comments to my "perplexolution" (EDGJ, Fall 1979, p. 62). The "perplexolution" consists of two congruent figures each defined by 5 vertices and 7 stright lines joining them. It is accompanied by a description of a procedure which permits the construction of all 5 vertices using a compass and a straightedge only, thus satisfying the traditional restriction in the choice of instruments. I admit to using, in addition to a compass, a parallel ruler and set-square. It is known, however, that tese extra instruments only help to make the process of drafting more efficient without violating the straightedge-compass restriction. Yet, because of the "No Calculations" condition in the original problem, the "perplexolution" has been declared by Pat as "not within the conditions of the puzzle". The exact meaning of the "No Calculations" condition is not known to me and it has not been defined in the problem. Pat explains in his letter that "if a solver has resorted to an algebraic calculation at any stage of the problem solving process then he has vilated the spirit of the 'no calculations' condition". Let us examine his own solution (EDGJ, Fall 1979, p. 60-61) in which he admits using "Abe Rotenberg's General Solution of any-pairs-of-non-Parallel-lines-appearing-equal-length". Rotenberg's "General Solution is based on a theorem makes use of the "three perpendiculars theorem". Readers are invited to check how the latter is proved in geometry textbooks. Here is one example reproduced from Solid Geometry by Hails & Hopkins, (Oxford University Press, 1957):

1.65. *If two lines be drawn from a given point outside a plane, one perpendicular to the plane and the other perpendicular to a line in the plane, then the line joining the feet of the perpendiculars is perpendicular to the line in the plane.*

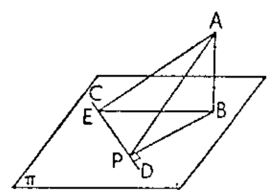


Fig. 12

Let AB be the perpendicular from the given point A to the plane π, meeting it in B (Fig. 12). Let P be the foot of the perpendicular from A to the given line CD in the plane; and let E be any other point of CD. Then

$$\angle APE = \angle ABP = \angle ABE \text{ (right angles)};$$

hence

$$\begin{aligned} BE^2 &= AE^2 - AB^2 \\ &= AP^2 + PE^2 - AB^2 \\ &= BP^2 + PE^2. \end{aligned}$$

Therefore $\angle BPE$ is a right angle.

I dare not make judgements whether this proof satisfies Pat's understanding of the "No Calculations" constraint. It seems reasonable to request the Editors of the EDGJ to explain to their readers the exact meaning of this constraint.

I believe that the calculations-vs.-constructions issue is important enough to every teacher of geometry and graphics to warrant the publication of this letter and to encourage your readers to express their opinions on this issue in your Journal.

Sincerely yours,

/s/ Abe Rotenberg.

We apologize for the delay in publishing Abe's open letter to the Readers of the Journal. The editors encourage more comments from the readers concerning the "calculation-vs.-construction" question. 'Tis a "knotty" problem. Should any algebra be used?? Your comments, please!!!! - Ed.

NOTE: The Journal invited Pat to comment. The following is his reply. Ed.

A. re: "I admit to . . . conditions of the puzzle."

Abe "admits to" using instruments but does not deny using a mathematical calculation in order FIRST to determine the numerical value of line length GD:

$$\sqrt{\frac{3-\sqrt{5}}{2}} \quad !!!$$

Since the puzzle conditions stipulated NO CALCULATIONS the 'Corner took the position that since Abe's solution depended upon the initial use of this calculation then his solution was not within the conditions of the puzzle. The corner stands by this position.

If, on the other hand, Abe will state that he intuitively (or in any other manner) knew this line length without a calculation then the 'Corner will acknowledge that it has done Abe and injustice.

B. re: "Let us examine . . . is a right angle."

I can only reply that I calculated nothing at any stage of the problem solving process. I used Abe's equal-lines-length general solution in conjunction with my own perpendicular-lines general solution, the latter being developed and used without benefit of my calculating anything. I assumed Abe's general solution was also, but in any case the USE of either general solution requires no calculation. The proof process, per se, (not the FACT that it has been proven) of his theorem or that of the textbook example which he cites is superfluous to the USE of the theorems themselves. To use a principle one only needs to know that it is true -- not necessarily how to prove that it is true.

C. re: "I dare not . . . 'No Calculations' constraint."

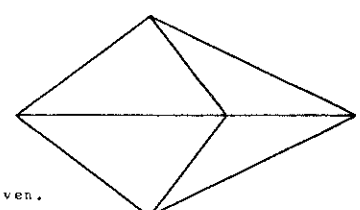
Here Abe seems to want me to apply the no-calculations condition to a PROOF! As the 'Corner's puzzle makes no mention of a proof, this analogy eludes us.

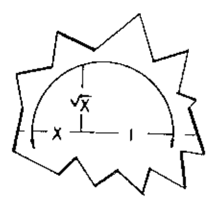
D. re; "It seems reasonable . . . this constraint."

Since the 'Corner was not responsible for the no-calculations constraint it does not feel competent to "explain" its meaning any further than has been done already. For this, interested readers are referred to the author of the puzzle, Bob Christenson.

R.P.K.

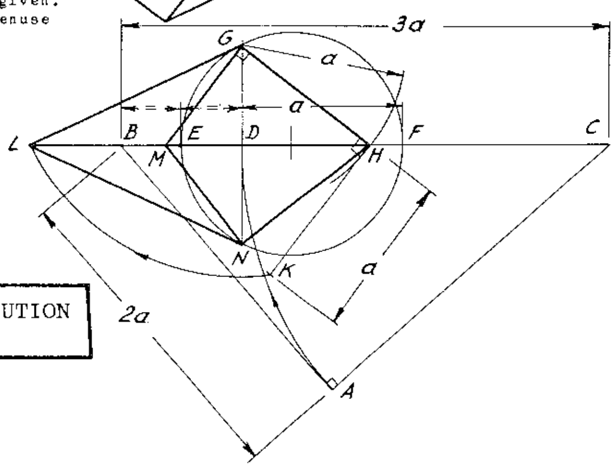
AFTERNOTE: Would you believe that Abe and Pat have been, are, and will continue to be fast friends? - Ed. (with the help of a crystal ball)





Let "a" be the side of each square of the "perplexahedron given.

1. Construct a right-angled △ABC with side AB=2a and hypotenuse BC=3a.
2. CD=AC
3. BE=ED
4. DF=a
5. DG ⊥ BC; G is on a circle with EF as diameter
6. GH= a
7. GM ⊥ CH
8. KH ⊥ GH; KH=a
9. GL=GK
10. DN=DG
11. The two principal views are congruent figures.



ABE ROTENBERG'S SOLUTION (FOR REFERENCE)

TEXAS A&M UNIVERSITY
DEPT. OF ENGINEERING DESIGN GRAPHICS
COLLEGE STATION, TX
APRIL 6, 1981.

Prof. Mary A. Jasper
P.O. Drawer HT
Mississippi State Univ.
Mississippi State, MS 39762

Dear Mary:

Congratulations on the winter issue of the Engineering Design Graphics Journal. This is a very good issue, and each seems to be better than the issue before.

I am writing to request that you continue our advertisement asking for applicants for teaching positions in our department. In case you don't have the latest notice, I am sending you a one page job description along with this letter.

Our university, and engineering in particular, has continued to grow at a very rapid rate and we are adding additional positions each year to keep pace with our growing enrollment. Consequently, we would like to see this ad run in every issue of the Journal to be sure that these positions are as widely advertised as possible.

Thank you for your assistance. Keep up the good work.

Sincerely,

/s/ James H. Earle, Head
Engineering Design Graphics

*Dr. Earle's job description can be found on Page **45**, This issue. The Journal encourages all departments which have positions available to send copy to the EDG Journal*
Drawer HT
Miss. State, MS 39762
We also appreciate Dr. Earle's kind comments concerning the present Journal. However, it is realized that there are still many editorial "hurdles" to be overcome, and any suggestions from the readers would be appreciated by the staff. - Ed.

1500 BAY ROAD, #415
MIAMI BEACH, FL 33139
APRIL 1, 1981

Toe (sic) the Editor:

"Lest We Forget the Goals" is one of the finest, most cogent and thoughtful articles I have ever read in the Journal. Paul DeJong has heightened my regard for his perspicacity and penetration to the heart of an increasingly cloudy mass of stuff called by the racy name "curriculum," Latin for a place to run. For years it has been a vicious circle, starting anywhere and ending in the same uncertain spot.

Descartes said it: "Cogito ergo sum." It means, "I think; therefore I exist." What better goal can education strive for than the ability to think? Anything else is unthinkable.

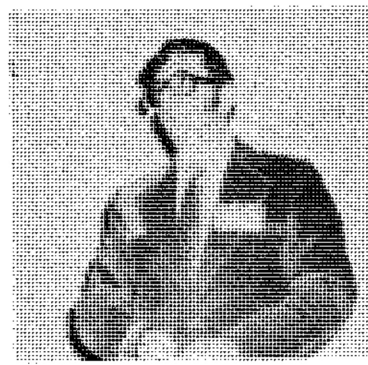
I think so. Congratulations to Professor DeJong.

/s/ Dr. Irwin Wladaver

I know that the salutation above is just a "Freudian slip", but the paranoid portion of my personality really believes that all of the readership would really like to use the whole "boot".
Nonetheless, we thank "Vlad" for his correspondence, his suggestions, but, most of all, for his wit. I think so, too. - Ed.

from the midyear conference

CONSULTING IN GRAPHICS -- GOOD FOR THE FACULTY AND GOOD FOR THE STUDENT



LARRY D. GOSS
ENGINEERING TECHNOLOGY
INDIANA STATE UNIVERSITY
EVANSVILLE, IN

As faculty, we are all in a financial bind. We're faced with small or non-existent salary increases for the foreseeable future; our spouses are working or trying to find work; we're faced with institutional restrictions on the type and amount of work we can pursue outside our teaching contracts; and we have the need to keep current in our disciplines. Our students have a similar list of needs. What I hope to do is to discuss a possibility to ease your situation and to help your students simultaneously. For each group, both for the faculty and for the student, we have two main goals:

1. Monetary Support - Your students are basically in school because they are after additional monetary support other than what they can get from flipping hamburgers at a fast food establishment or being a supervisor of a department at a discount house.

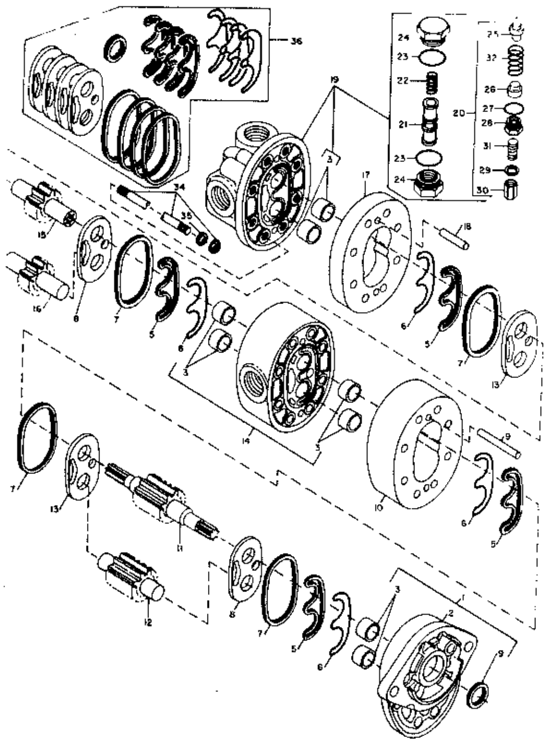
2. They need some experience in the field in which they are endeavoring to enter.

But what I want to do first is to review for you some of the classical means we have for support, and indicate what some of the problems and some of the benefits of each of those are. This may not be all inclusive, but it will allow you to review your own thinking of what is available for students and also for faculty with respect to additional support during the academic year.

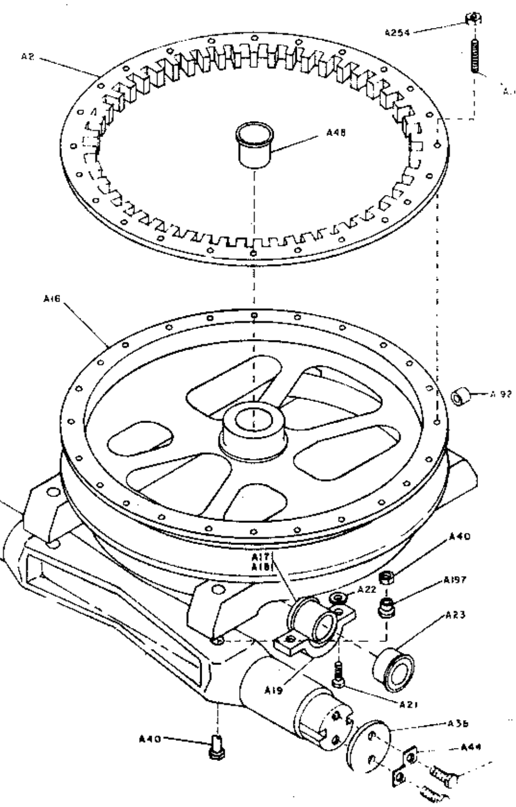
The first of these and probably the most classic of all time is the summer job. The summer job has some very definite pitfalls and lack of benefit. It has little or no coordination with the engineering curriculum. There is no help or recognition for the institution, and obviously there is no faculty involved in this method of student support.

Work-Study Program--I did not actually hear about work-study myself until I became acquainted with institutions which did not have graduate programs. But, there are a number of institutions around the country which do not have graduate programs and cannot rely upon graduate assistants for all of their "gopher-type" work situations. As a result they have work-study programs. These are primarily federally-funded programs--a kind of a scholarship for the individual student. Eligibility for these programs is unrelated to the students' skills; it is more related to their economic status. The compensation is minimal and most job tasks are at very low levels. Faculty involvement between the student and the faculty member is very minimal.

Scholarships. These basically end up being a reward for past performance. A scholarship is frequently awarded on the basis of the student's grade point average. There are no industry-related skills or experiences involved for the reward of most scholarships, no directly-related learning experiences, and faculty involvement is questionable.



RIGHT HAND PUMP



Student Design Projects. Advantages of the design project are that generally it has good high-level design experience involved with it, but it leaves the student with an unrealistic view about what really goes on in industry. They see industry from a different perspective than they would if they were employees of that particular company. It has good faculty involvement because usually faculty are involved with input to the design projects that are industrially sponsored. But generally speaking, it is part of a course that is already being offered as part of the regular contract that amounts no additional renumeration for the faculty member.

Institutionally supported Work Programs. Once again, pay is generally low, learning is not necessarily related to the discipline of the student and faculty involvement may or may not occur.

Cooperative Education. This activity contains good industrially related experience. Students actually see what a company looks like inside because they do real work for that company. However the type of experiences they have vary with the understanding that the institution and the industry have with respect to what cooperative education is suppose to be. I've seen some programs where the involvement by the student and the level of work they do are good. I've also seen instances in which the individual was in a "make-work" situation. There is also a problem with cooperative education that frequently the student goes back to the same job with the same company on a second and third co-op term and he ends up doing the same tasks over and over again. Co-op students do get reasonable pay. Once again, however, there is little or no faculty involvement in cooperative education. The involvement may be such that the faculty member does make an onsite visit to the student once during the term of cooperative education, but there is no real interchange between the student and the faculty member.

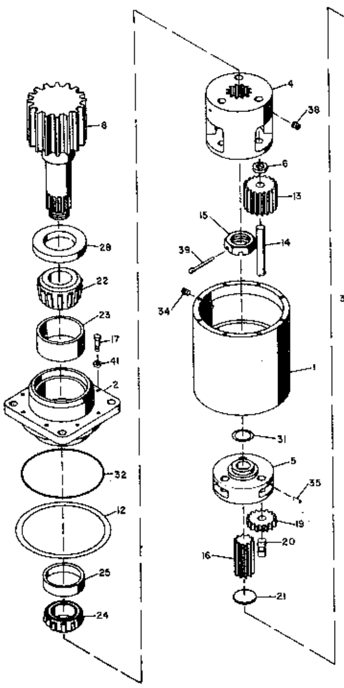
Consulting Contracts. With consulting contracts, the student and the faculty member get good industrially related experience; the pay is reasonable, and there is an intense direct involvement between students and faculty members.

What type of consulting contracts am I talking about? What are our students doing? Our students have had contracts to work with the faculty on such diverse projects as:

Surveying--Everything from soccer fields to land holdings for corporations.

Destructive Testing--Concrete samples, reinforcing bars, mine roof bolt systems.

Planimetry--Coal mine reserve calculations.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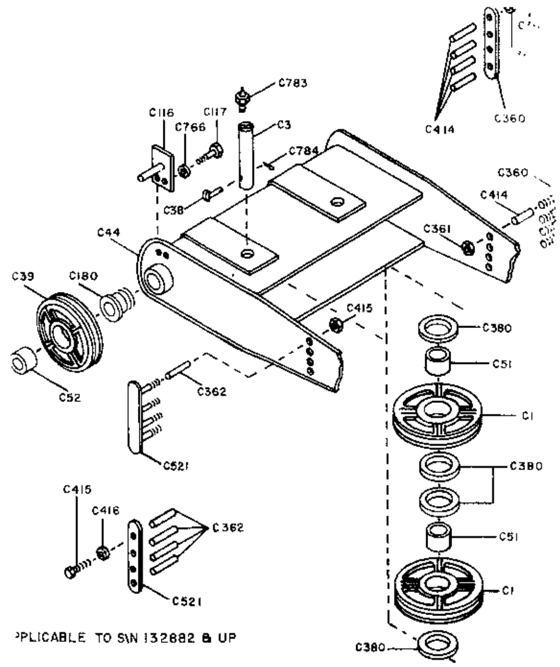
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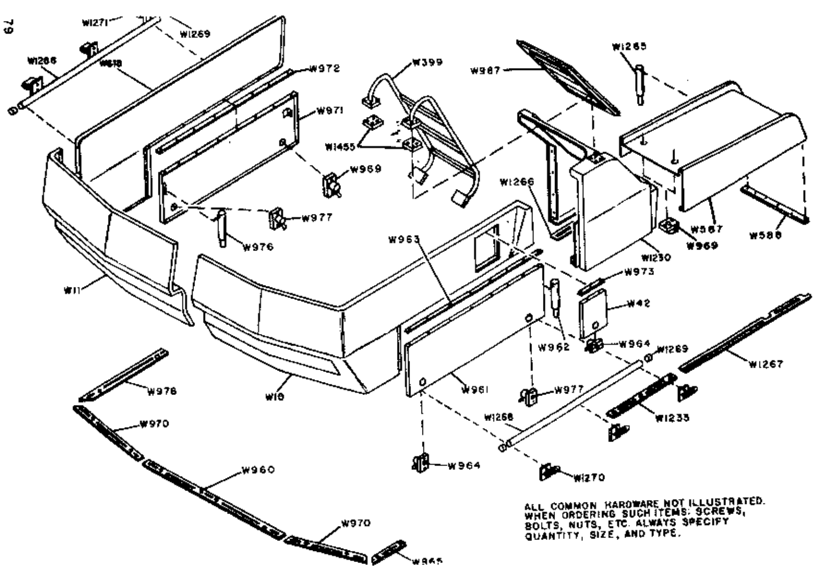
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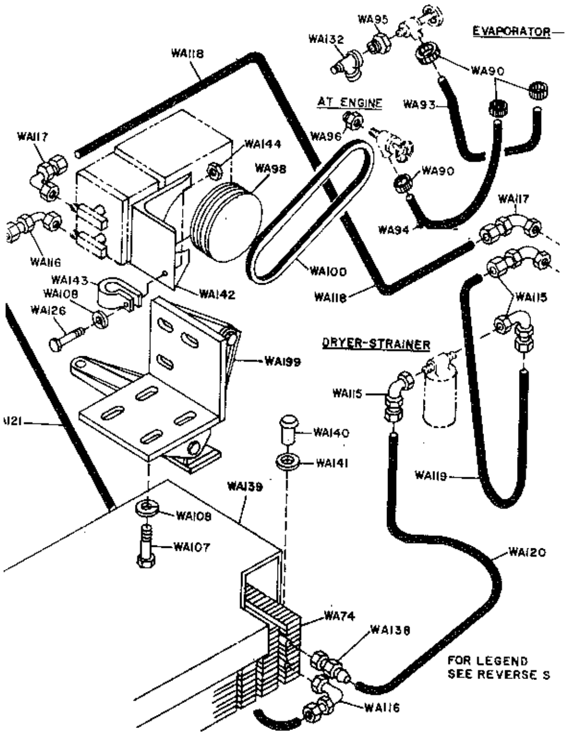
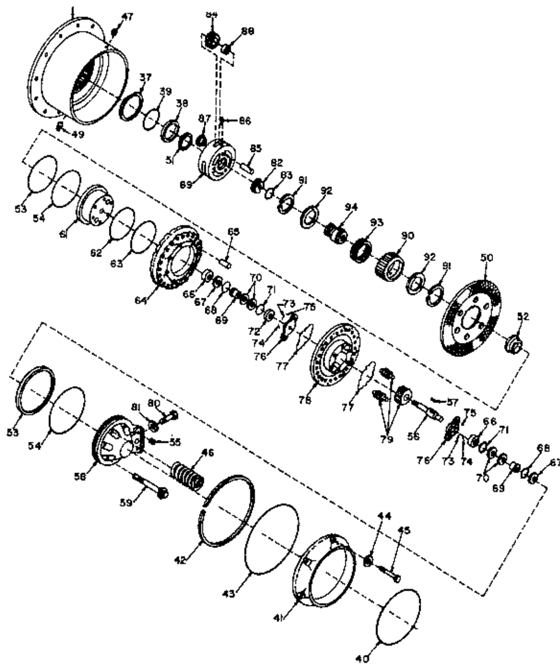
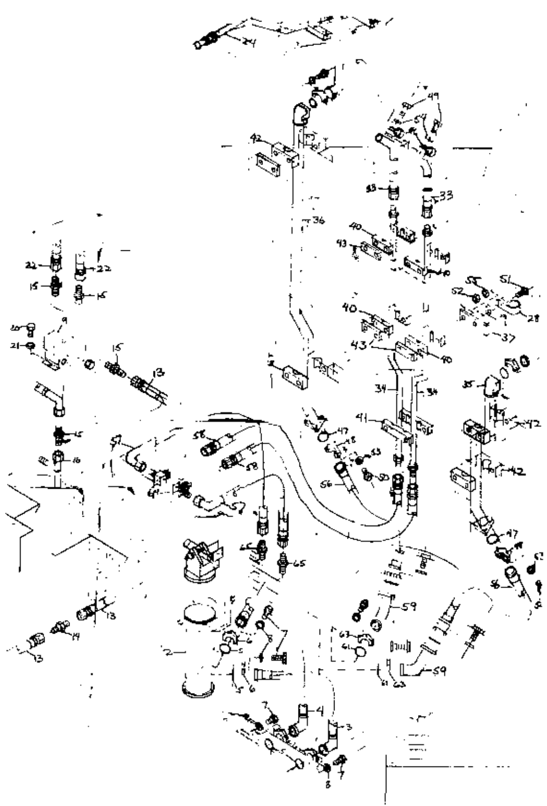
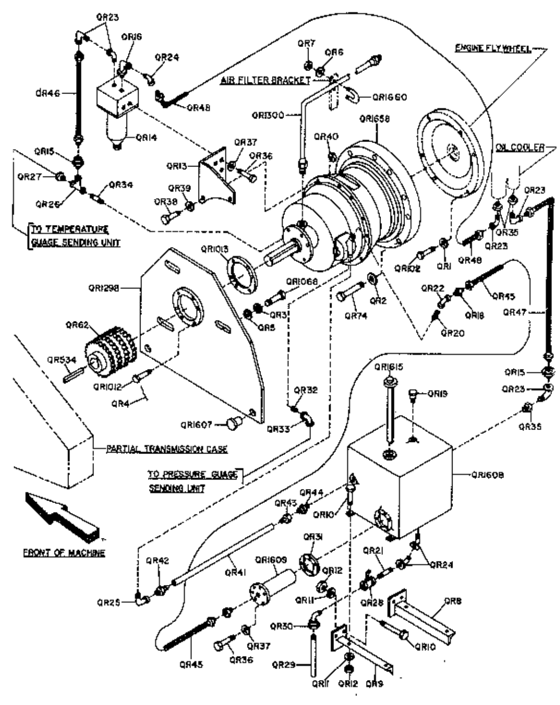
ALL COMMON HARDWARE NOT ILLUSTRATED.
WHEN ORDERING SUCH ITEMS: SCREWS,
BOLTS, NUTS, ETC. ALWAYS SPECIFY
QUANTITY, SIZE, AND TYPE.



HEATER & AIR CONDITIONING PIPING







Sampling--Traffic counts, transportation utilization surveys, etc.,

Technical Illustration--Illustrated parts breakdown catalogs.

The work accomplished by freshman and sophomore students in this last category should be of particular interest to readers of the EDG Journal. Each of the figures accompanying this article is an example of the illustrations completed by students who were hired by a company on a consultant basis to execute camera-ready artwork for inclusion in the company's parts catalogs.

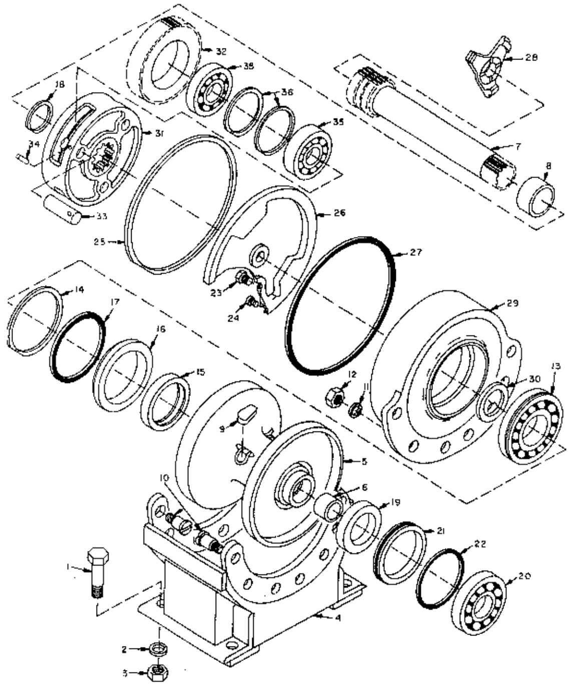
Each of the students was able to compile a sizeable portfolio of work during the seven months he or she was involved with this work, and each received immeasurable experience in handling media and in learning drafting techniques that can not be imparted during the confines of a studio or laboratory course of instruction. In addition, the monetary rewards to the student and faculty member were exceptionally good. The faculty member gains in supervising such consulting work by remaining current in his or her field and by maintaining industrial contacts.

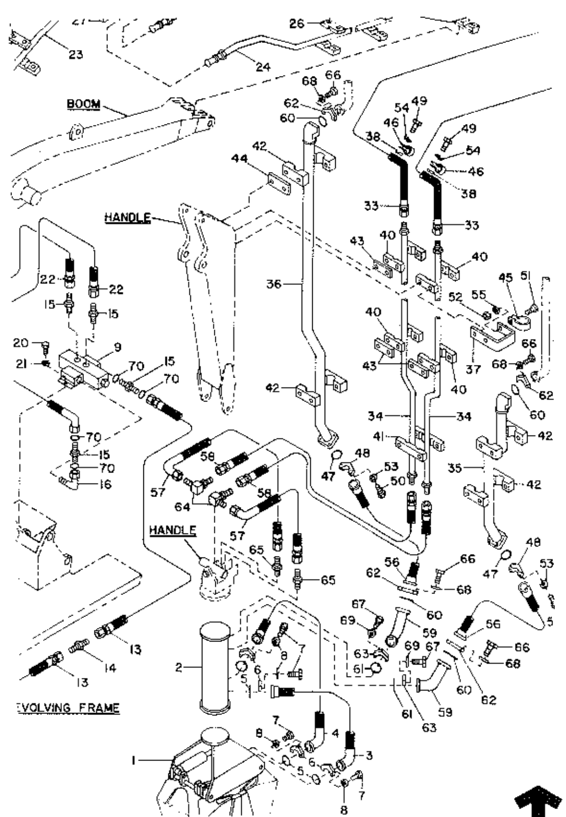
How do you get the work? One method is to use contacts in your industrial advisory board. Another is to make use of industrial contacts you may already have. A third method is to trust to pure luck--wait for your telephone to ring. I use all of these methods, but the method that has worked best is an agent relationship that I have established with a local printing company that services industries in my immediate area with the printing of parts catalogs.

The benefits of consulting contracts can be summarized as follows:

For the students; they receive a good monetary reward for the time spent, the work is under the direct supervision of a faculty member, and the industry gets a look at their developing talent without a long-term commitment.

For the faculty; it allows you to keep current and maintain industrial contacts, and it allows you to augment your teaching contract so that your income is on a par with engineers in industry.





ASEE - EDGD PERSONAL

TIME	MONDAY JUNE 22	TUESDAY JUNE 23
Before 8:00 a.m.	**1138 Judging Criteria for Creative Design Display** **7:30-9:45 a.m.** **COM B** **Breakfast $5.50** **ENGINEERING DESIGN GRAPHICS DIVISION** **Moderator:** Robert Foster, Pennsylvania State University Organizational meeting for judges of the Creative Design Display.	
8:00 a.m. to 9:45 a.m.	**1238 Teaching Techniques in Graphics** **8:00–9:45 a.m.** **VKC 156** **Lecture** **ENGINEERING DESIGN GRAPHICS DIVISION** **Moderator:** Retha Groom, Texas A&M University **Speakers:** *Computerized Testing for Freshman Graphics Course*—Lia Brillhart, Triton College *Use of Patents as a Teaching Technique*—Robert S. Lang, Northeastern University *Compnter Graphics as an Aide in Beginning Design Projects*—Roy Hartman, Texas A&M University	**2238 Women in Engineering: A Status Report** **8:00-9:45 a.m.** **GER 124 Aud.** **Lecture** **ENGINEERING DESIGN GRAPHICS DIVISON** **Co-sponsors:** Engineeriug Design Graphics-Freshmau Engineering Committee, Relations with Industry Division-Women's Action Group, Women in Engineering Committee **Moderator:** Blaine Butler, Purdue University **Speakers:** *Women in Engineering: An Overview*—Ada Pressman, Society of Women Engineers *Resource People for Women in Engineering*—Lois Greenfield, University of Wisconsin *Pre-College Career Awareness*—Joe Moeller and S. Schwartz, Stevens Institute of Technology *Performance Data on Women in Engineering*—William LeBold and Carolyn Jagacinski, Purdue University *Observations from a Woman in Engineering*—Emily Maddox, Dupont Company
10:00 a.m. to 11:45 a.m.		
12:00 Noon to 1:30 p.m.		
1:45 p.m. to 3:30 p.m.		**2546 Affordable CAD/CAM Systems and Software** **1:45-3:30 p.m.** **VKC 156** **Symposium** **ENGINEERING TECHNOLOGY DIVISION** **Co-sponsors:** Computers in Education, Engineering Design Graphics and Relations with Industry Divisions **Moderator:** John B. VanSaun, Wichita State Univeraity Inexpensive alternatives to commercial CAD/CAM systems are available. Academic and industrial users will describe some of the choices. Types of systems to be discussed will include: smart graphics terminals, desk top computer/calculators and home-computer based systems. **Speakers:** *A Performance Comparison of Less Expensive Turnkey Systems*—Ron Frank, Purdue University–Indianapolis *Development of a Home-Computer Based System for Basic CAD/CAM Instruction*—Kay S. Mortensen, Brigham Young University *Effective Utilization of Inexpensiva Computer Graphics Systems*—Gerry Greiss, Eastern Michigan University
3:45 p.m. to 5:30 p.m.	**1838 Needs of Computer Graphics Staff in Industry** **3:45-5:30 p.m.** **VKC 156** **Lecture** **ENGINEERING DESIGN GRAPHICS DIVISION** **Co-sponsors:** Computers in Education and Engineering Technology Divisions **Moderator:** Robert D. Harvey, College of DuPsge The growth of computer-aided design and manufacturing in industry has crested needs for both prsctitioners and developers of the methods and systems. The selection of systems, hardware and software; the development of new capabilities in design and analysis; and the training of senior designers and supervisors will be addressed. **Speakers:** *The University's Role in Meeting Industry's CAD/CAM Needs*—Mark Shepard, Rensselaer Polytechnic Institute *Integrating Computer Graphics into Technical Training Programs*—Victor Langer, Milwaukee Area Technical College	
Dinners	**1738 Engineering Design Graphics Executive Committee Dinner** **6:00 p.m.** **UHH 214** **Dinner $15.50** **ENGINEERING DESIGN GRAPHICS DIVISION** **Moderator:** Paul DeJong, Iowa State University	**2738 Engineering Design Graphics Division—Awards Banquet** **6:00 p.m.** **UHH 1880** **Dinner $15.50** **ENGINEERING DESIGN GRAPHICS DIVISION** **Moderator:** Paul DeJong, Iowa State University 6:00 p.m. Social Hour. 7:00 p.m. Dinner.

WEDNESDAY
JUNE 24

CREATIVE DESIGN DISPLAY

Engineering Design Graphics Division Creative Engineering Design Display

The fourteenth annual display of student design projects, sponsored by the Engineering Design Graphics Division, will be located in the Shrine.

Monday	**12:00 noon-5:00 p.m.**
Tuesday	**8:00 a.m.-5:00 p.m.**
Wednesday	**8:00 a.m.-1:00 p.m.**

Design entries are grouped by academic year and are judged for awards. Judges are selected from edncation and industry. Each year, this display has attracted strong participation and support from industry. Companies supporting past awards have inclnded: Amoco, Sverdrup/ ARO, Inc., Boeing Aircraft, The Celanese Corporation, Eastman Kodak Company, E.I. dnPont deNemonrs Company, Ford Motor Company, General Motors Company, Monsanto Company, Olin Corporation, and Union Carbide Corporation.

Display Committee: Jay S. Abramowitz, Indiana-Purdne University, Robert J. Foster, Pennsylvania State University, Jon K. Jensen, Marquette University, Edward Knoblock, University of Wisconsin-Milwaukee.

Faculty interested in entering design projects may receive details from:
Robert J. Foster
245 Hammond Building
Pennsylvania State University
University Park, PA 16802
(814) 865-2952

3238 Human Factors in Engineering Education

8:00-9:45 a.m. **SAL 127** **Lecture**

ENGINEERING DESIGN GRAPHICS DIVISION
Co-sponsor: Indnstrial Engineering Division
Moderator: John Kreifeldt, Tufts University

This session will bring together industrial designers, the Human Factors Society and educational leaders to define the function of human factors engineering, the desired content of HFE education and the response of engineering education to these needs.

Speakers:
Interrelation of Industrial Design and Human Factors—Gene Garfinkle, Design Group
The Education and Employment of Human Factors Engineers—Robert Bescoe, Professional Improvement Company
Engineering Education's Response to the Need for Human Factors Engineers—Hal Hendrick, Wheeler Air Force Base

3438 Engineering Design Graphics Division Luncheon

12:00-1:30 p.m. **DCC 221** **Luncheon**

ENGINEERING DESIGN GRAPHICS DIVISION
Moderator: Paul DeJong, Iowa State University

3538 How Geometric Dimensioning and Tolerancing Can Save Industry Money

1:45-3:30 p.m. **SAL 101 Aud.** **Lecture**

ENGINEERING DESIGN GRAPHICS DIVISION
Co-sponsor: Engineering Technology Division
Moderator: Betty Prescott, San Joaquin Delta College

Why industry wants graduate engineers and technologists to understand geometric dimensioning and metric practices.

Speakers:
How Y 14.5 Can Save Industry Money—George Tokunaga, Lawrence Livermore Laboratory
Metric Practines and Dimensioning—Edward Mochel, University of Virginia

3588 Pre-College Engineering Programs for Women and Minorities

1:45-3:30 p.m. **SAL 127** **Symposium**

RELATIONS WITH INDUSTRY-WOMEN'S ACTION GROUP
Co-sponsors: Engineering Design Graphics Freshman Engineering and Women in Engineering Committees
Moderator: Jane Daniels, Purdue University

Directors of successful pre-college engineering programs for women and minorities will present anecdotal or longitudinal studies of their programs. Emphasis will focus on descriptions of the programs themselves and data resulting from subsequeut evaluations. Anyone involved in pre-college programs or wanting to become iuvolved is encouraged to attend.

Speakers:
Pre-College Awareness Programs—Joe Moeller, aud Susan Schwartz, Steveus Institute of Technology
Influences affecting the Career Choice of High School Girls—Maxine McCurnin, California State University-Long Beach
Pre-College Engineering Programs for the Indentification and Motiviation of Minority Students—Marion Blalock, Pnrdue University
1971-1980: Longitudinal Studies of Pre-College Programs for Women and Minorities in Engineering—Marilyn Burman, University of Maryland

3695 Engineering-Academia or Industry?

3:45-5:30 p.m. **DCC** **Symposium**

WOMEN IN ENGINEERING COMMITTEE, SOCIETY OF WOMEN ENGINEERS
Co-sponsors: Engineering Design Graphics and Liberal Studies Divisions, Relations with Industry Divisiou-Women's Action Group
Moderator: Margaret Ellers, Louisiana State University

This is a joint meeting of the ASEE Women in Engineering Committee and the Society of Women Engineers. Long range planning discussions between ASEE members interested in women in engineering and SWE members will take place during a social hour. Conclusions reached will be summarized in a position paper or proceedings.

THE INNER WORLD OF THE PLANE-- FORM BASES FOR PICTURES, FILMS, AND "VISUAL MUSIC"

REINHARD LEHNERT
NORDALLEE 12
6638 DILLINGEN/SAAR
FEDERAL REPUBLIC OF GERMANY

> "If, in her lifeless beginnings Natur had not been so completely stereo-metric, how could she wish finally to achieve incalculable and infinite life?"
> Goethe: Wilhelm Meisters Wanderjahre

This article resumes the subject of three earlier articles in this Journal (Winter 1975 cover and pp. 7-18; Fall 1978, pp. 42-54; Winter 1980 pp. 48-61) and of several articles in other magazines (cf. list of publications in the article of Fall 1978). As its predecessors, this article deals with geometric primitive forms (the "inner-stars") discovered by the author, and pictures designed on these forms as form-bases (the "inner-pictures"), and films (the "inner-games"), and thus with a basis for a "visual music", an art form which offers something analogous and equivalent to the eye as music is to the ear. In it the inner-stars are to play a role such as the scales "play" in music. The present article deals especially with inner-stars of the "square point grid of the number of extension 3" (sP3) with the following attribute:

> The border segments of the "base-figures" have more complicated gradients. More exactly, the gradients are 2, -2, $\frac{1}{2}$ and $-\frac{1}{2}$.

In addition, this article offers an outlook on inner-stars of sP3 with still more "complicated" gradients.

1. The Form-Sources of Pictorial Art

As discussed in the previous article, pictorial art obtains its forms from three great "form-sources":

A. The world of geometric forms.
B. The world of representative forms and forms serving a purpose.
C. Accident.

The aesthetic and artistic sense performs the selection, the deformation and the combination of geometric, representative and accidental partial forms achieving aesthetic and artistic total forms.

2. The Form-Bases of Pictorial Art

The artists construct their works on definite form-bases. We distinguish such form-bases as:

a. Objective.
b. Objective-subjective
c. Subjective.

The "classical" schools are founded on objective form-bases. The "modern" schools are founded on objective-subjective and subjective form-bases. Exemplary of objective art are; idealistic, realistic, naturalistic graphic, painting, plastic, dramatic and film-art: examples of objective-subjective art are; static constructivism (ornaments, Malevich, Mondrian, Kandinski in his geometrical pictures, Vasarely and others), dynamic constructivism (constructivistic visual kinetics including the constructivistic film not representing objects),

Picture 1.

surrealism and kindred artistic schools: and, representing totally subjective form-bases, tachism and kindred artistic schools.

3. "Visual Music" and Constructivism

"Visual music" and constructivism both use the geometric form-base, primarily. They differ from each other in that "visual music" uses objective form-bases; constructivism, on the other hand, uses objective-subjective form-bases. Thus, only the "visual music" is an optical parallel to the art of music which also uses objective form-bases (viz. the scales and their laws). Only "visual music" is able to offer something analogous and equivalent to the eye as music offers to the ear.

However, every visual-kinetic game a and every non-representative film is also isolated. They have nothing in common; there is no relationship, no tradition, and no development -- no lasting resonance with the public. As far as I know, there are no works of these arts that could be compared with the works of Bach, Mozart and Beehoven or even of lesser composers, in their artistic content.

4. About the Objectivity of the "Inner-Stars"

The inner-stars differ from all other possible plane form-bases by possessing the following attributes:

1. They are "homogenous" (in layered approximation); i.e., they are equally constituted in all places, thus equally structured.

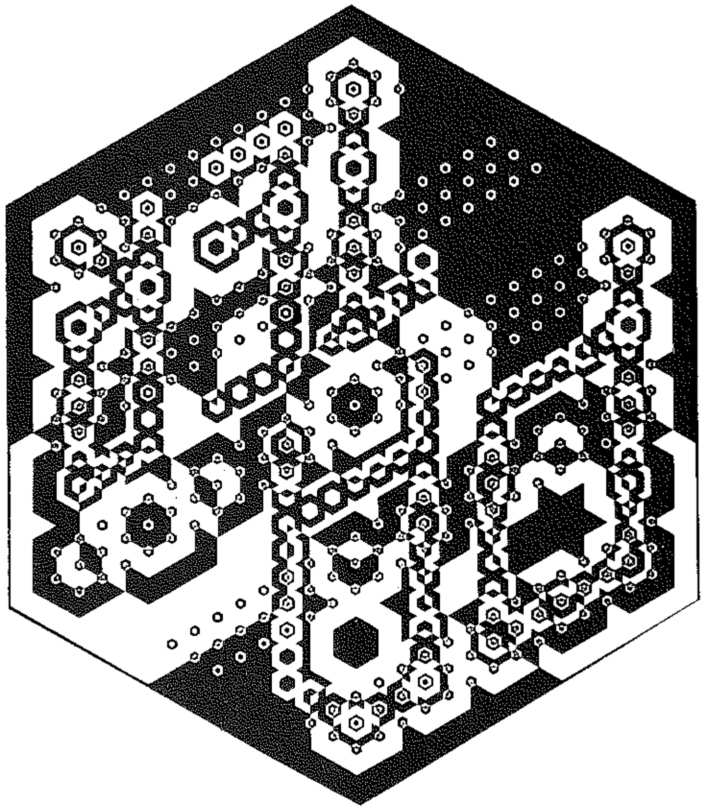
2. They are integrally structured in a hierarchial manner, i.e., they arouse the impression of an arranged entity.

Homogeneousness is the fundamental attribute of space and of the plane, (this attribute also applies to time and space-time, also), in classical physics: the same geometry is applied to all space. The entireness is a fundamental attribute of the work of art.

5. Picture 1.

Picture 1 is a "seven-circle-picture". It was designed on the "reduced layered seven-circle" in the following way:1)a circle is drawn, which we call the "circle of layer 0";2)seven circles are "imbedded" in this circle, which we call the"circles of layer 1", and the radii of these circles are 1/3 of the radius of the circle of layer 0 -- further, the uppermost of these circles touches the circle of layer 0 at its top point; 3)in each of these circles of layer 1 we draw 7 circles of layer 2 in the same manner; etc.

Picture 1 presents: 1)the circle of layer 0 (that is the largest circle with its border line coincident with the borderline of the picture; 2)3 circles of



Picture 2.

1 (they are the second largest circles); 3)9x2=18 circles of layer 2 (they form two hooked figures forming angles of 120°); 4)18x7=126 circles of layer 3 (they form 2x7=14 such hooked figures which, compared with those of layer 1 are turned around to the right by 60°; 5)162x7=1134 circles of layer 4 (they form 14x7=98 hooked figures which compared with the preceding figures are turned round to the right by 60° again.)

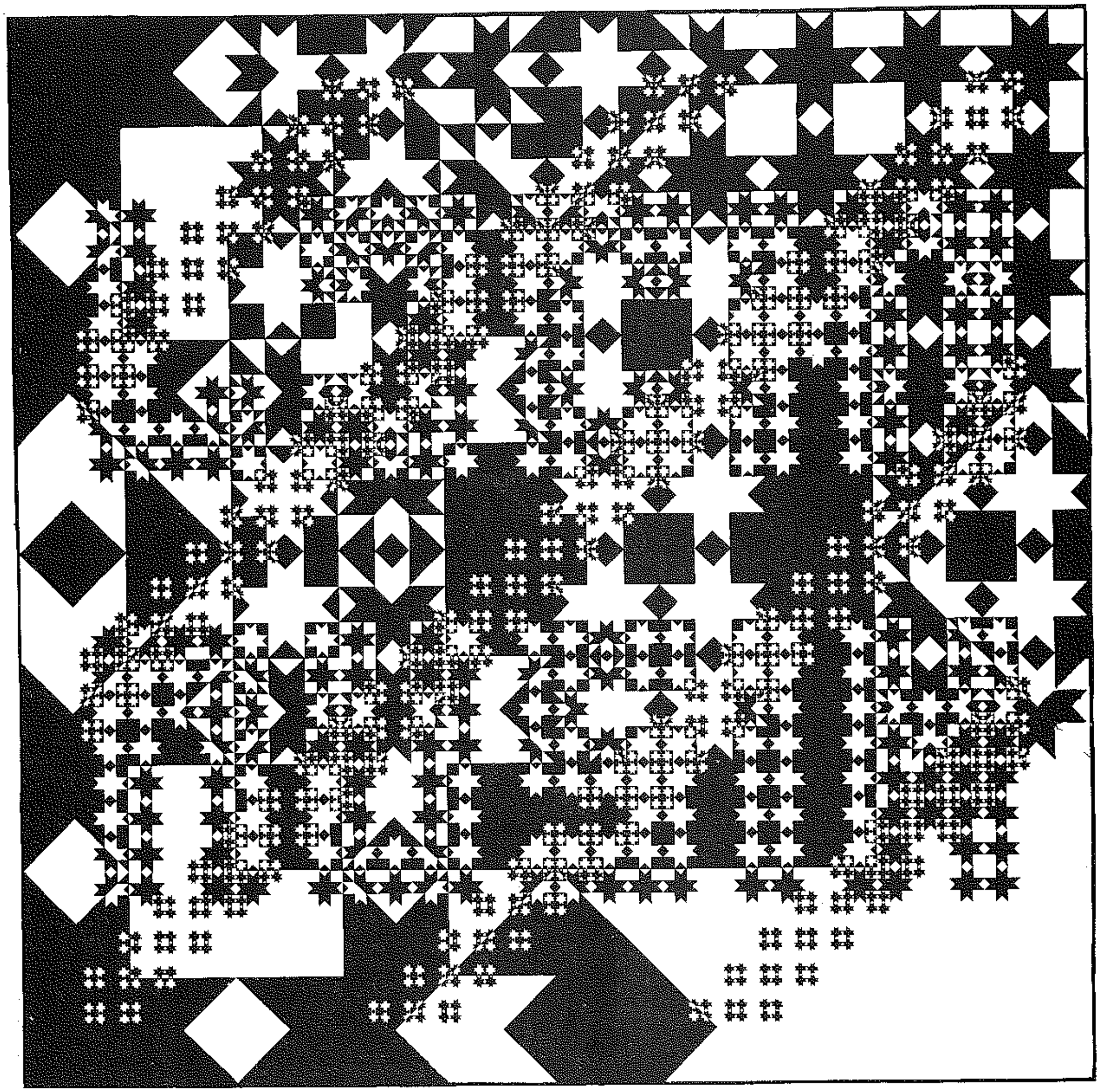
6. Picture 2.

Picture 2 is an "inner-picture", more exactly, a "fusion-picture". It was designed on an inner-star of the "hexagon point-grid of the number of extension 3"(hP3). The base-figures are regular octagonal rings. Each base-figure overlaps each of its six neighbors in a rhomb. It is compulsory that such neighbors, when they appear in the same picture, "fuse" with each other. The outer and inner border line of such a "base figure of layer n" is composed of 6 "strong-segments of layer (n+1)" each.

Picture 2 shows: 1)3 base-figures of layer 1 (top left, in the middle and top right); 2)14+16=30 base-figures of layer 2 (they form a "P" and an "A"; 3) 33+33+15+19+24=124 base-figures of layer 3 (they form the stylized letters "PA-TER"); 4) 45+63+79+63+45=295 base-figures of layer 4 (they form 3x5=15 ornamental rows).

7. Picture 3.

Picture 3 is an "inner-picture", more exactly, a "linked-picture". It was designed on an inner-star of the "square point-grid of the number of extension 3", or sP3. The base-figures are "rings" consisting of four stars with eight in--dentations each. Each base-figure covers each of its four neighbors in two of these stars. It is "linked" with its neighbor by these two stars. In addition, it also covers its four "half-neighbors" top right, top left, below right, and below left, and that in one of each of these stars. The

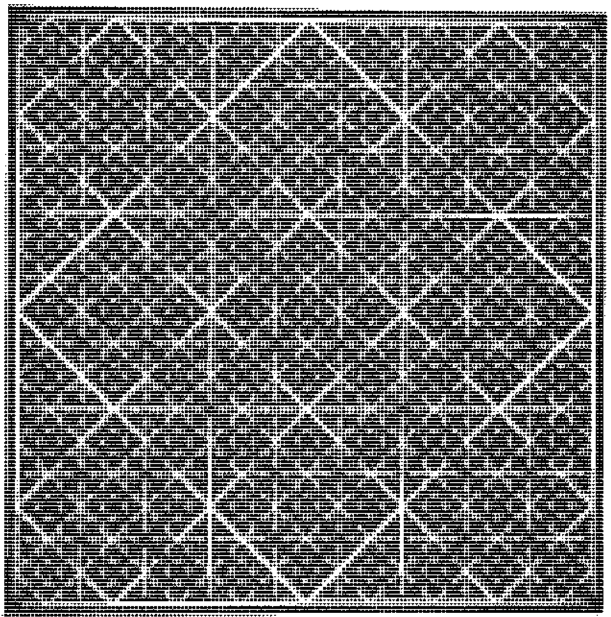


Picture 3.

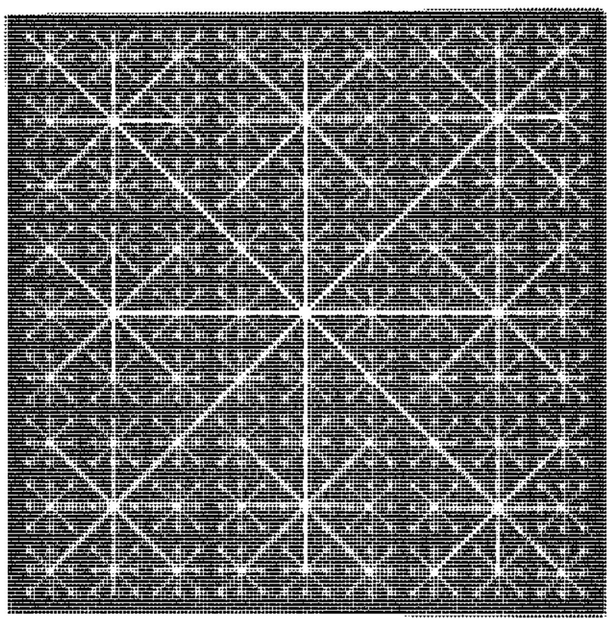
border line of a base-figure of layer n is composed of 12x4=48 strong-segments of the next finer layer (n+1) each.

Picture 3 shows:1)base-figure of layer 0; 2)3 base-figures of layer 1 (on the left side of the picture); 3)22 base-figures of layer 2 (they form an "M"); 4)139 of layer 3 (they form the stylized letters "LICHT-MUSIK'); 5)241 of layer 4 (they form 5 ornamental rows).

Picture 4.



Picture 5.



8. Picture 4 and "cell-star" sC3.

By its vertical and its horizontal lines, picture 4 presents the "square cell-star of the number of extension 3", or sC3. More exactly it shows: 1)its central "cell of layer 0" (that is the square picture itself, conceived as the plane-piece; 2)its partitioning into 9 "cells of layer 1" (these are the second largest squares); 3)their partition into 9 "cells of layer 2" each (these are the third largest squares); 4)their partition into 9 "cells of layer 3" each (these are the smallest squares).

It is to be remembered that the whole sC3 is obtained if we: a)add the cells of layers 4, 5, 6, . . .; b)repeat the figures obtained in this way beyond the borders of the picture in the arrangement of an infinite regular net of squares until the whole plane of the picture is covered; c)also add the "cells" of the coarser layers -1, -2, -3, . . . to the total figure obtaned in this manner.

9. Picture 5 and the "point-star" sP3.

Correspondingly by its star-figures Picture 5 presents the "square point-star of the number of extension 3", or sP3. More exactly, it shows the central cell of layer 0 and in it the "strong-points" of layers 0, 1, 2 and 3. At this point, we should remember that we obtain the whole sP3 if we: a)add the "strong-points" of layers 4, 5, 6, . . . ; b)repeat the figure thus obtained beyond the borders of the picture in the arrangement of an infinite regular net of squares until the whole plane of the picture is covered; c)also add the "strong-points" of the coarser layers -1, -2, -3, . . . to the total figure thus obtained.

We recognize, as in the former articles that for each n out of the series . . ., -3, -2, -1, 0, 1, 2, 3, . . . applies: 1)the "cells of layer n" form a regular square parqueting of the plane of the picture; 2)the "strong-points of layer n" form a regular square point-grid in the plane of the picture; 3)the "strong-points" of layer n are the centres of the "cells of layer n"; 4)each strong-point of a layer n coincides with a strong-point of each of all the finer layers (n+1), (n+2), (n+3), . . .; 5)in the center of the picture and only there lies one strong-point of every layer; 6)a centric reduction in the measure of 1:(1/3) maps the cells and the strong-points of layer n on the ones of the next finer layer (n+1), thus it maps the total sC3 and the total sP3.

10. Pictures 4 and 5, the "strong-" and "field lines".

For the inner portion of the central cell of layer 0 and for the gradients ∞ (gradient angle 90°), 0 (0°), 1(45°), -1(-45°), picture 5 shows all the strong-lines of layers 0, 1, 2, and 3. We remember: 1)"strong-lines of layer n" are all the straight lines going through at least two (and through infinitely many) "strong-points of layer n"; 2)"field-lines of layer n and the number of division f" are straight lines dividing field stripes between two neighboring strong-lines of layer n into f equal stripes; f can be every natural number which is relatively prime to the number of extension e of the underlying point-star, in the case in question -- therefore, e=3; 3/every field-line of a layer n and the number of division f coincides with one field-line of each of all finer layers (n+1), (n+2), (n+3), . . . to the same number of division f. We call every partial line segment of a strong-line a "strong-segment", of a field line a "field-segment".

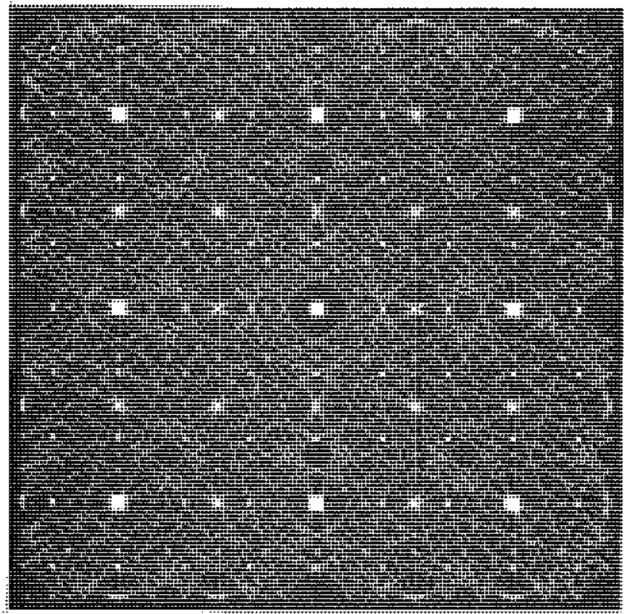
11. Introduction of coordinates.

We introduce Cartesian coordinates; the zero of the coordinate system is the center of the picture; the axis of x points to the right; the axis of y points to the top; the unity of length is the lateral length of thepicture, hence the lateral length of a "cell of layer 0". Next, the strong-points of layer 0 are the points (a;b), and and b being any integer; the strong-points of layer 1 are the points ((a/3);(b/3)); those of layer 2 are the points ((a/9),(b/9)); those of layer 3 are the points (a/27); (b/27))etc.; in general: those of layer n are the points $((a/3);(b/3^n))$. The corner points of the picture and therewith of the central cell of layer 0 are the points ((1/2);(1/2)), -(1/2);(1/2)) and ((1/2);-(1/2)).

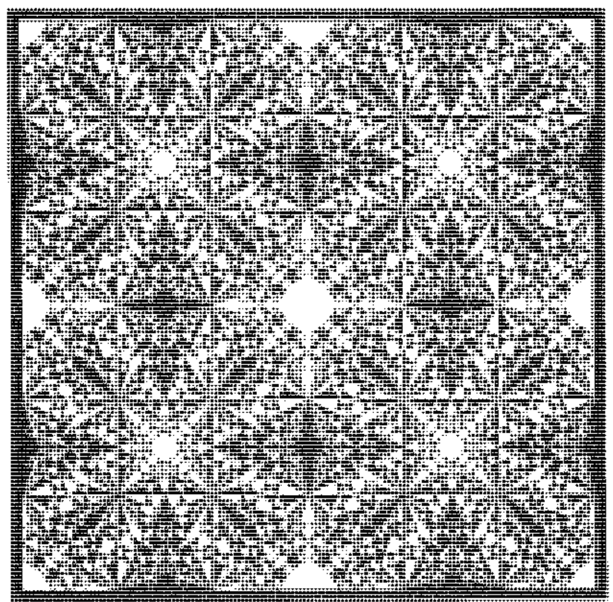
12. Picture 6, its "complete-star" and its "universal-star".

Picture 6 likewise shows the central cell of layer 0 of sP3. More exactly, it shows 1 strong-point of layer 0, 3x3=9 strong-points of layer 1, and 9x9=81 of layer 2 by small squares appearing massively white. For the interior of the central cell of layer 0, it shows, furthermore, all "free stronglines of layer 1" in so far as they have the gradients ∞, 0, 1, -1, 2, -2,(1/2), -(1/2), 3, -3, (1/3), -(1/3), (3/2), -(3/2), (2/3), -(2/3). These are those among the lines shown. We remember: A strong-point of a layer n is called "bound" when it coincides with a strong-point of the next coarser layer (n+1), otherwise "free"; the same applies for strong-lines and field-lines, strong-segments and field-segments.

As it is evident, picture 6 shows 2 strong- and 2 field-lines each for the gradients ∞ and 0; 4 strong- and 4 field-lines each for the gradients 1 and -1; 6 strong- and 6 field-lines each for the gradients 2, -2, (1/2), and -(1/2); 8 strong- and 8 field-lines each for the gradients 3, -3, (1/3), and -(1/3); 10 strong- and 10 field-lines each for the gradients (3/2), -(3/2), (2/3), and -(2/3). Thus picture 6 shows layer 1 of a "complete-star" for the interior of the central cell of layer 0. We remember: a "complete-star" consists of all strong- and field-lines of definite (and that only finitely many) gradients and a definite number of division f that are coordinated to a point-star; the complete stars serve to design "inner-stars" easily and, above all, systematically; a "universal-star" comes into being from



Picture 6.

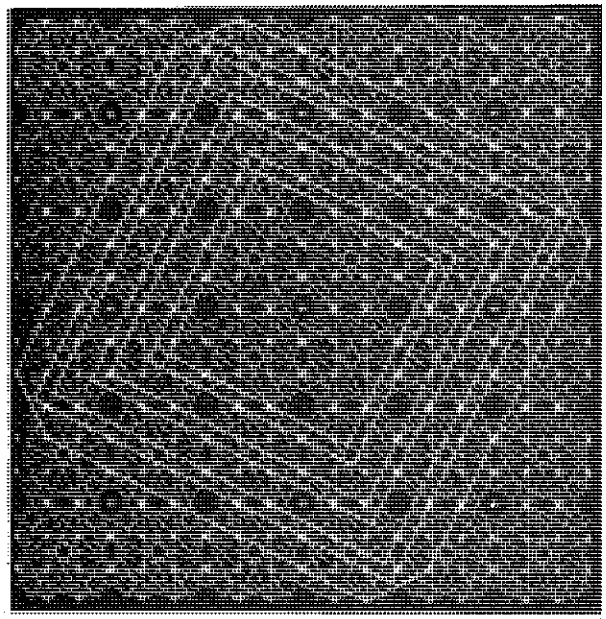


Picture 7.

a "complete-star" by adding all the "bound" strong- and field-lines co-ordinated to this one; complete-stars and universal-stars are straight-line-complexes, i. e. figures composed of (infinitely many) straight-lines.

13. Picture 7

Picture 7 shows the figure of picture 6 once more, now however with its plane-pieces colored in black and white. It reminds us in a faint way of "Laue-diagrams" that come into being when crystals are treated with x-rays. That also seems comprehensible when we consider that the "point-stars" are geometrically related to the crystals: In every layer

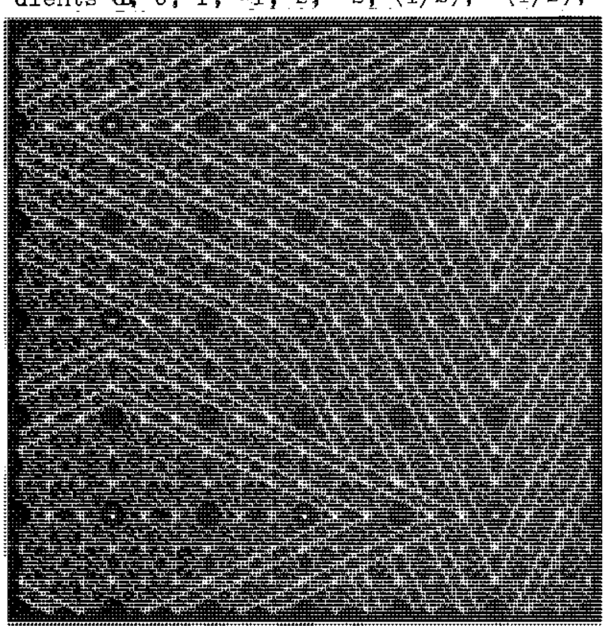


Picture 8.

they show a "two dimensional grid of crystals". Each of the infinitely many complete-stars results in the described manner in a geometrical picture in black and white. Certainly obtaining such pictures is only an unimportant by-product of our efforts about inner-stars, inner-pictures and inner-games.

14. Pictures 8-11 and their "inner-stars".

Just as pictures 6 and 7, the pictures 8-11 show the interior of the central cell of layer 0 of sP3. In this cell they show: 1. The strong-points of layer 1; 2. the free strong-lines of layer 2, in so far as they have the gradients ∞, 0, 1, -1, 2, -2, (1/2), -(1/2);



Picture 10.

3. the free field-lines of layer 2 in so far as they have the same gradients and belong to the number of division each. --- We remember: An inner-star of sP3 comes into being from the cell-star sC3 by variations of the same kind of all "cells" whereby new plane-pieces coherent in themselves; B. The border lines of two base-figures each of different (!) layers have only finitely many points in common with each other (We call this attribute of the inner-stars their "transparence"); C. The base-figure are divided by the border-lines of the base-figures of all respectively coarser layers in only a finite number of ways (We call this attribute of the inner-stars their quality of being joined in themselves with regard to the layers; D. Every base-figure has at least one point each in common with its four neighbors upwards, downwards, to the right, and to the left (We call this attribute of the inner-stars their "connection in every layer").

15. Evidence of the attributes A-D of the inner-stars.

For each of the border-lines presented in the pictures 8-11 the following applies: 1. It shall limit a base-figure of layer 1; 2. It is designed on the net of strong-lines and field-lines of layer 2 shown in the pictures 8-11; 3. Therfore it is composed of strong-segments and field-segments of layer 2; 4. From this follows that the total figure produced by it possesses the attributes B and C; 5. Moreover it is designed in such a way that the total figure produced by it also possesses the attributes A and D.

16. Evidence of an additional attribute E.

The border-lines shown in the pictures 8-11 are chosen in such a manner that they have no partial line segments in common with the border-lines of the co-ordinated base-figures of the same (!) layer. We call inner-stars whose base-figures have this attribute E "transparent with regard to base-figures of the same layer". This attribute is achieved in the cases in question by the partial line segments of the border-lines shown not exceeding a certain "critical length". For the partial line-segments with the gradients ∞ and 0 this critical length is the distance from andy strong-point of layer 1 to its neighbor upwards, downwards, to the right or to the left, e. g. from point (0;0) to point (0;(1/3)). For the partial line-segments with the gradients 2, -2, (1/2), and -(1/2) this "critical length" is the distance from the lower central strong-point of layer 1 to the upper right strong-point of layer 1, hence from point (0;-(1/3)) to point ((1/3);(1/3)).

17. "Mixed" inner-pictures and -games.

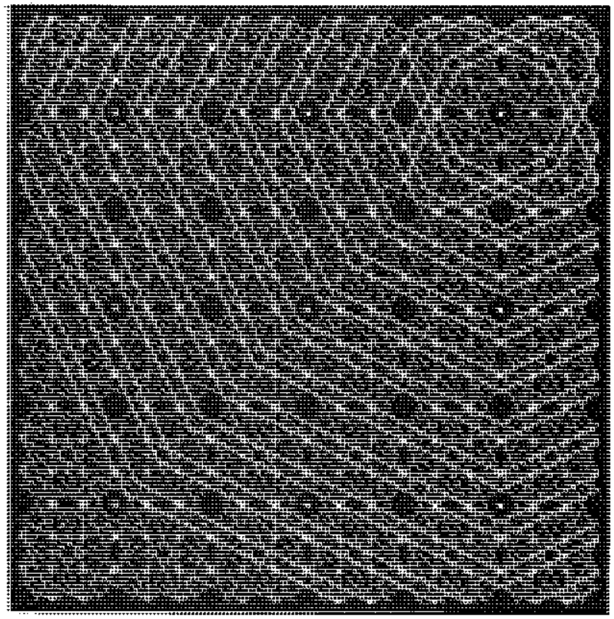
We can make the base-figures of two or several inner-stars of the same point-star appear "conjointly" without any difficulties, i.e. in the same inner-picture, if these base-figures do not "disturb" each other, more exactly if their border-lines do not cover each other. In this case we call the inner-stars in question "compatible" with each other. Several inner-stars I_1, I_2 etc. are certainly "compatible" with each other, e. g., if the border-segments of the base-figures of I_1 have definite gradients, those of the base-figures of I_2 different gradients, those of the base-figures of I_3 again different gradients etc. For instance this is for three inner-stars I_1, I_2, I_3 the case when the border lines of I_1 have the gradients ∞, 0, 1, and -1 only, those of I_2 the gradients 2,-2, (1/2), and -(1/2) only, those of I_3 only the gradients 3, -3, (1/3) and -(1/3). Two inner-stars I_1 and I_2, e.g., are certainly "compatible: with each other, too, when the border segments of the base-figures of I_1 are strong-segments and those of the base-figures of I_2 are field-segments.

18. Picture 8

Picture 8 presents 7 base-figures of layer 1 which we designate with the numbers 8;1-8;7, beginning with the smallest. Each of these base-figures overlaps its four neighbors, the appertaining inner-star is therefore an "overlapping-star". The base-figure of its kind: all the still smaller ones produce total figures having the above mentioned attributes A, B, and C, indeed, but not the attribute D. 8;6 is the largest possible base-figure of its kind which still has the attribute E. Therefore we exactly get 6 inner-stars of the described kind if we demand additional attribute E. 8;7 is a slanting square with its angles cut off. These angles were cut off in order to grant the attribute E to the co-ordinated inner-star.

The border-lines of 8;2 and 8;3, 8;6 and 8;7 are composed of strong-segments, those of 8;1, 8;4 and 8;5 of field-segments. Consequently each of the inner-stars of 8;2, 8;3, 8;6 and 8;7 is "compatible" with each of the inner-stars of 8;1 and 8;5. 8;3 distinguishes itself by its corner-points being strong-points of layer 2. The inner star of 8;3 is the inner-star of pictures 31;3 and 32.

The border segments of the base-figures 8;1-8;6 only show the gradients 2 and (1/2). A reflexion at the axis of y results in six new base-figures which we designate by the numbers 8;1'-8;6'. Their border segments only show the gradients (1/2) and -2. Consequently each of the inner-stars 8;1-8;6 is "compatible" with each of the inner-stars 8;1'-8;6'.

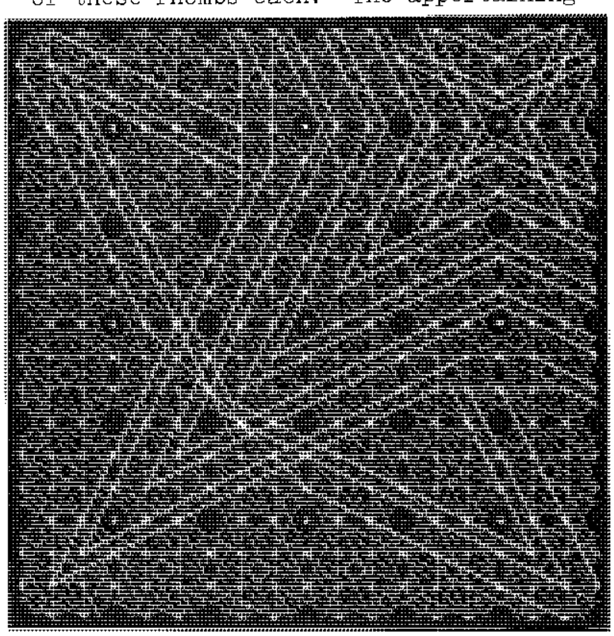


Picture 9.

We get further kindred inner-stars if we also admit slanting squares lying excentrically to the strong-points represented by them. We get further ones if we also admit (concentric or eccentric) slanting rectangles as base-figures, or parallelograms with partly oblique partly vertical or horizontal sides.

19. Picture 9.

Picture 9 presents 22 base-figures of layer 1 which we designate with the numbers 9;1-9;22, beginning with the smallest. 9;1 and 9;2 are octogons with four attached rhombs each. These base-figures cover their four neighbors upwards, downwards, right and left in one of these rhombs each. The appertaining



Picture 11.

inner-stars are, consequently "linked-stars". The figures 9;1 and 9;2, furthermore, represent a further "linked-star" each. Their base-figures are "rings" consisting of four octogons each partly overlapping each other which have the form of stars. 9;3-9;22 are octogons overlapping their four neighbors. The appertaining inner-stars are consequently "overlapping-stars".

The border-lines of 9;1, 9;4 and 9;5, 9;8 and 9;9, 9;12 and 9;13, 9;16 and9;17, 9;20 and9;21 are composed of strong-segments, those of 9;2 and 9;3, 9;6 and 9;7, 9;10 and 9;11, 9;14 and 9;15, 9;18 and 9;19, 9;22 of field-segments. 9;1 and 9;4, 9;9 and 9;12, 9;17 and 9;20 distinguish themselves by possessing corner-points (exactly four) being strong-points of layer 2. 9;3 is the smallest possible base-figure of its kind. 9;22 is the largest possible base-figure having the attribute E.

The base-figures 9;7-9;14 cover the centres of the neighboring base-figures in the directions upwards, downwards, to the right and to the left. 9;15-9;22 cover moreover the centres of the neighbors in the directions upwards-upwards, downwards-downwards, right-right, left-left. The reader may examine the covering of the centers of the remaining "neighbors of higher degrees".

The base-figures 9;3-9;8 come into being from the base-figures 8;1-8;6 of picture 8 by production of symmetry which is achieved by cutting off four angles each. The second linked-star co-ordinated to the figure 9;1 is the inner-star of picture 30;2. The inner-stars of the base-figures 9;5-9;8 are those of the pictures 20-23. The inner-stars of the base-figure 9;9 is that of picture 31;1. We obtain numerous further centric base-figures resembling circles, especially larger ones than 9;22 if we admit "octogons with their angles out off" whereby the angles are cut off by strong-segments or field-segments of the gradients ∞, 0, 1 and -1.

20. Picture 10/downwards-left

Picture 10/downwards-left presents four base-figures of layer 1 which we designate with the numbers 10d;1-10d;4, beginning with the smallest. They are rhombs whose vertices point to the right and to the left. By their form they emphasize the horizontal direction. They produce overlapping-stars. Their border-lines only have the two gradients (1/2) and -(1/2). 10d;1 is the smallest possible base-figure of its kind. 10d;4 is the largest possible base-figure of its kind possessing the attribute E. The border-lines of 10d;2 and 10d;3 are composed of strong-segments, those of 10d;1 and 10d;4 of field-segments. All four corner points of 10d;2 are strong-points of layer 2. Only two corner points of 10d;3, namely the vertices, are strong-points of layer 2.

A turning by 90° produces four new possible base-figures which we designate with the numbers 10d;1'-10d;4'. Their vertices point upwards and downwards. Consequently they emphasize the vertical direction. Their border-lines only have the gradients 2 and -2. Therefore each of the inner-stars of the base-figures 10d;1-10d;4 is "compatible" with each of the base-figures 10d;1'-10d;4'.

21. Picture 10/top-right.

Picture 10 presents 16 base-figures of layer 1 as the center. Beginning with the smallest we designate them with the numbers 10u;1-10u;16. 10u;1 and 10u;2 are stars with four rhombs each attached at the vertices. The appertaining inner-stars are linked-stars. 10u;3-10u;12 are octogons, and that stars with four vertices pointing upwards, downwards, to the right and to the left. The appertaining inner-stars are overlapping-stars. 10u;3 is the smallest possible base-figure of its kind, 10u;12 is the largest possible base-figure of its kind fullfilling the condition E. 10u;13-10u;16 are dodecagons whose border lines also contain strong- and field-segments of the gradients 1 and -1 in addition, 10u;16 is the largest possible base-figure of its kind: all the larger ones contain border-segments exceeding the appertaining "critical length". The appertaining inner-stars are overlapping-stars. The border-lines of 10u;2 and 10u;3, 10u;6 and 10u;7, 10u;10 and 10u;11, 10u;14 and 10u;15 are composed of strong-segments of layer 2, those of 10u;1, 10u;4 and 10u;5, 10u;8 and 10u;9, 10u;12 and 10u;13, 10u;16 of field-segments of layer 2. Figure 10u;4 produces the overlapping-stàr of picture 30;1, and in different interpretations the adjacent-star of picture 30;3 and the linked-star of picture 30;4.

22. Picture 11/center

Around the center of the picture, picture 11 presents 3 base-figures of layer 1 which we designate with the numbers 11c;1-11c;3, beginning with the smallest. 11c;1 and 11c;2 are rhombs whose vertices point upwards-left and downwards-right. By their form they emphasize the direction with the gradient -1, that is the direction of the "second" diagonal of the square of the picture. They produce overlapping-stars. Their border-segments have only the two gradients -2 and -(1/2). 11c;1 is the smallest possible base-figure of its kind. 11c;2 is the largest possible one of its kind still having the attribute E. The border-line of 11c;2 is composed of strong-segments, that of 11c;1 of field-segments. Only two corner points of 11c;2, and that the vertices, are strong-points of layer 2. 11c;3 is a hexagon whose border lines are composed of strong-segments of the gradients -2 and -(1/2) and of field-segments of the gradient -1. 11c;3 is the largest possible base-figure of its kind still having the attribute E.

A turning by 90° results in three new possible base-figures with the numbers 11c;1'-11c;3'. By their form they emphasize the direction with the gradient 1, hence the direction of the "first' diagonal of the square of the picture. Their border segments only show the gradients 2 and (1/2), respectively 2, (1/2) and 1. Therefore each of the inner-stars of 11c;1-11c;3 is "compatible" with each of the inner-stars of 11c;1'-11c;3'.

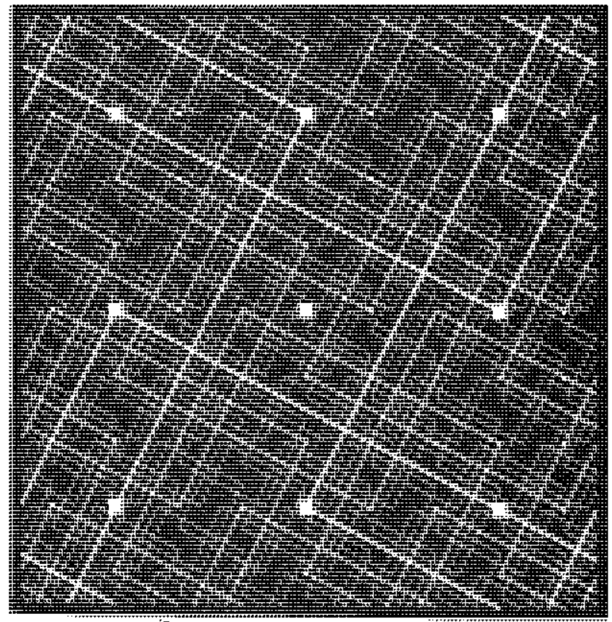
21. Picture 11/top-right

Around the upper right strong-point of layer 1 as the center, picture 11 presents 10 base-figures of layer 1 which we designate with the numbers 11u;1-11u;10. 11u;1 and 11u;2 produce the inner-stars of the pictures 17 and 18. As it may be observed from these two pictures, 11u;1 and 11u;2 may be interpreted as rings consisting of four hexagons each as well as stars with four further stars attached at the vertices. 11u;3-11u;8 are octogons, and that stars with four vertices each pointing upwards-right, upwards-left, downwards-right, and downwards-left. The appertaining inner-stars are overlapping-stars. 11u;3 is the smallest possible base-figure of its kind. 11u;10 is the largest possible one of its kind still having the attribute E. 11u;9 and 11u;10 are dodecagons whose border lines also contain field- and strong-segments of the gradients ∞ and 0. 11u;10 is the largest possible base-figure of its kind: all the still larger ones contain border-segments exceeding the appertaining "critical lenght". The appertaining inner-stars are overlapping-stars.

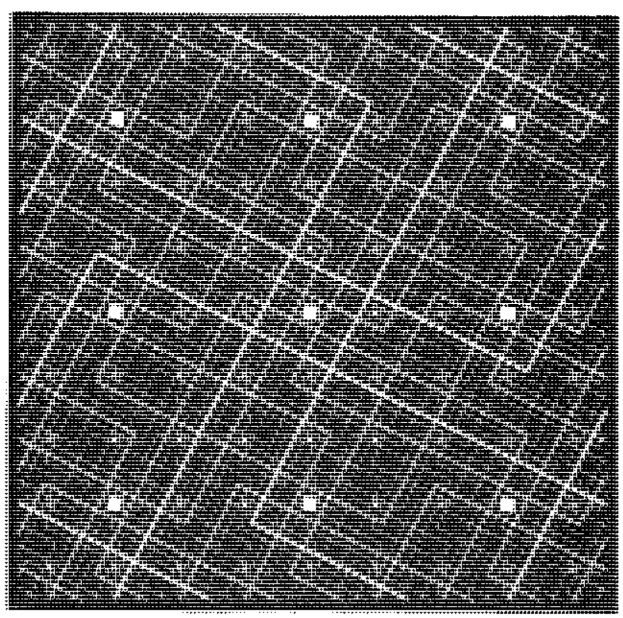
The border-lines of the base-figures of 11u;2 and 11u;3, 11u;6 and 11u;7, 11u;10 are composed of strong-segments of layer 2, those of 11u;1, 11u;4 and 11u;5, 11u;8 and 11u;9 are composed of field-segments of layer 2. Therefore each of the five inner-stars mentioned first is "compatible" with each of the five mentioned last. All 8 corner points of 11u;3 and 11u;6 and all 12 corner points of 11u;10 are strong-points of layer 2.

24. Pictures 12-27.

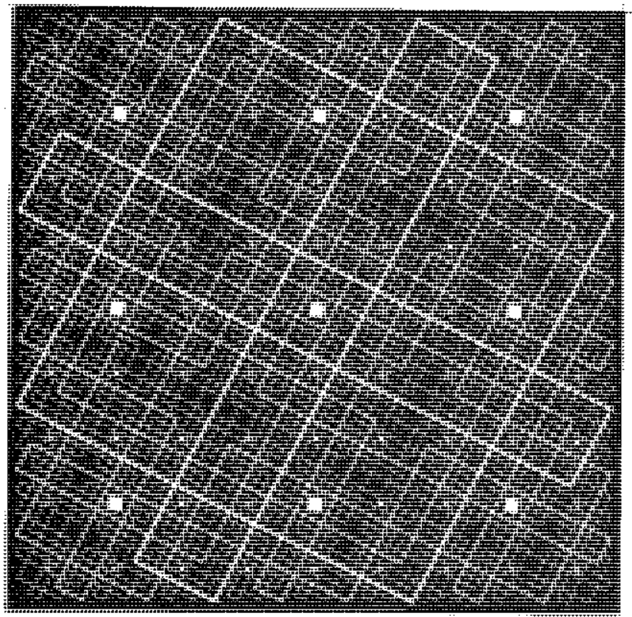
The pictures 12-15 present the inner-stars of the pictures 8;3-8;6. More exactly: of these inner-stars they show the interior of the cell of layer 0 and in it the base-figures of the layers 0, 1, and 2. The base-figures are slanting squares. The border lines of the base-figures of 12 and 15 are composed of strong-segments, those of the base-figures of 13 and 14 of field-segments. These border segments become longer from pictures 12 to 15 at every picture, and you recognize at once when looking at these pictures that they cannot grow longer than in picture 15 if it is to have the attribute E. As mentioned above, the inner-star of picture 12 is that of the pictures 31;3 and 32.



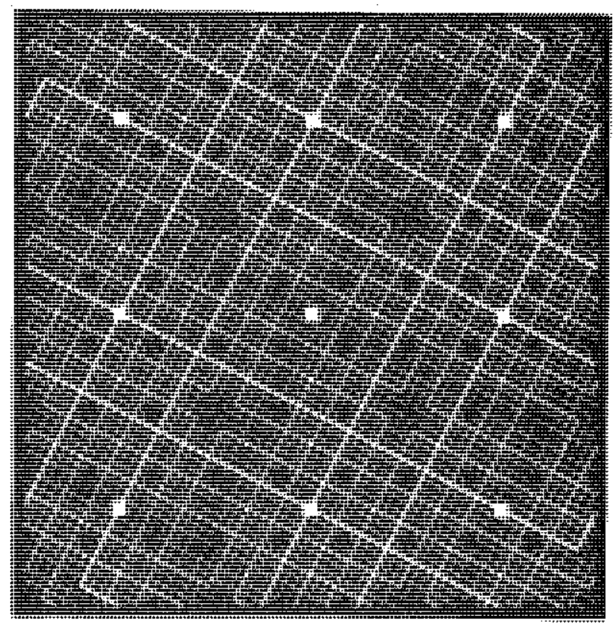
Picture 12.



Picture 13.



Picture 14.

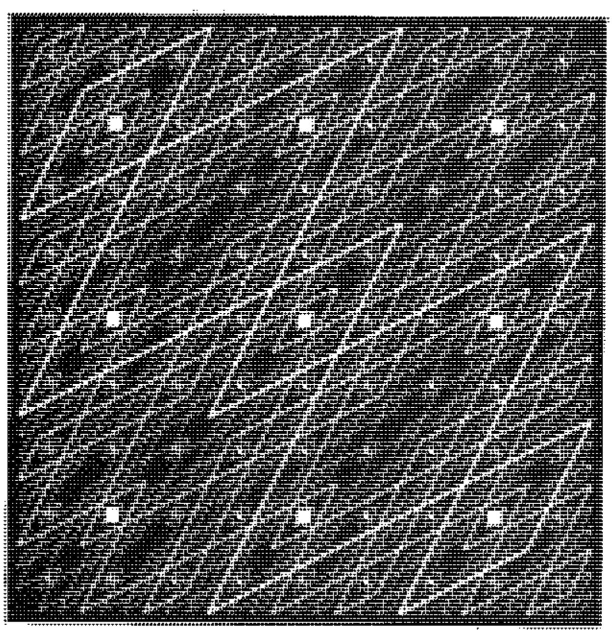


Picture 15.

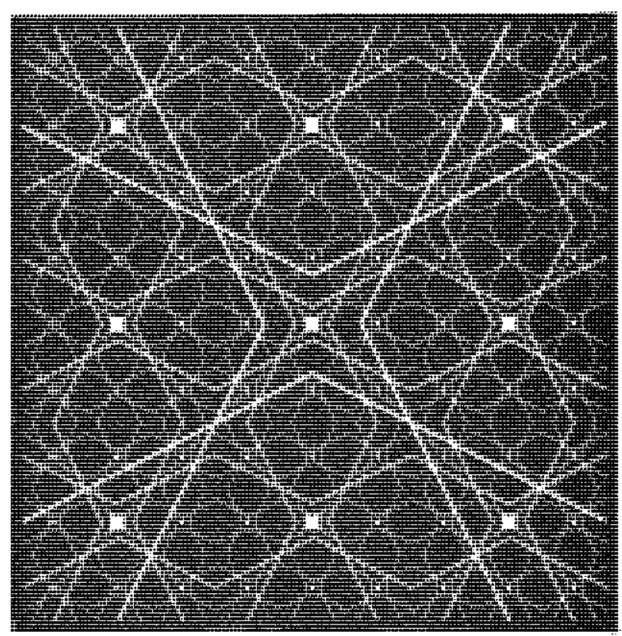
The pictures 16 and 19 present the inner-stars of the pictures 11c;1' and 11c;2' in the same way. When looking at the pictures 16 and 19 you recognize at once that these two inner-stars are the only possible ones of their kind having the attribute E. In the same way the pictures 17 and 18 present the two inner-stars of the pictures 11u;1 and 11u;2 mentioned above (No. 23).

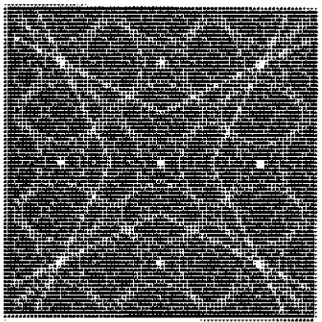
In the same way, the pictures 20-23 present the inner-stars of the pictures 9;5-9;8; whereas the pictures 24-27 present the inner-stars of the pictures 10d;1'-10d;4'. When looking at the pictures 24-27 you recognize at once that these four inner-stars are the only possible ones having the attribute E.

Picture 16.

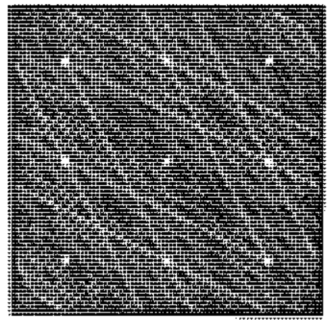


Picture 17.

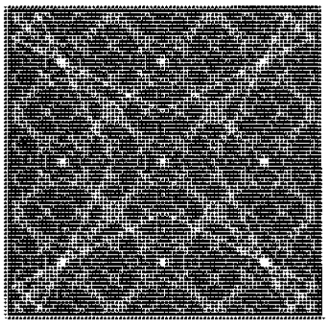




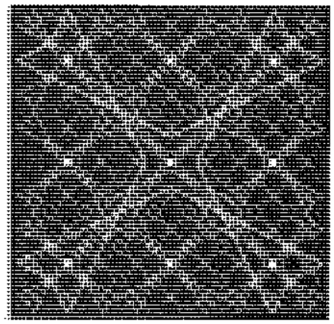
Picture 18.



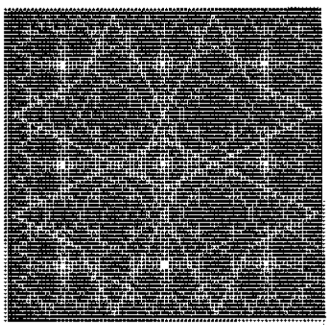
Picture 19.



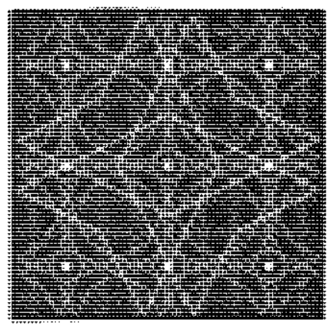
Picture 20.



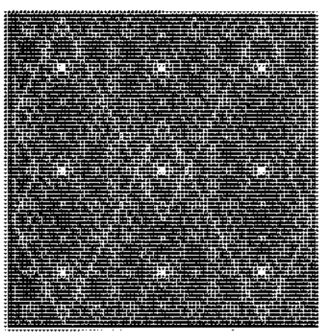
Picture 21.



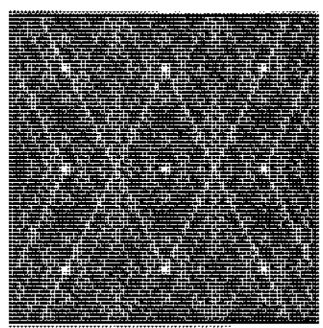
Picture 22.



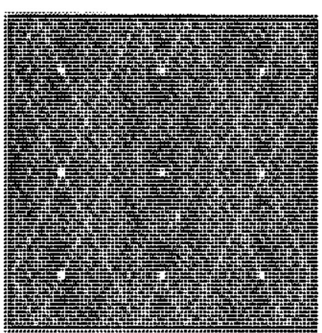
Picture 23.



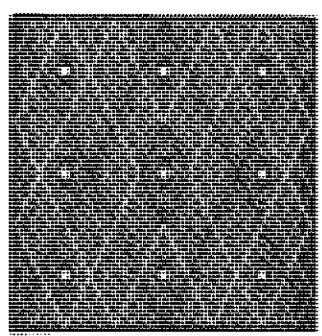
Picture 24.



Picture 25.



Picture 26.



Picture 27.

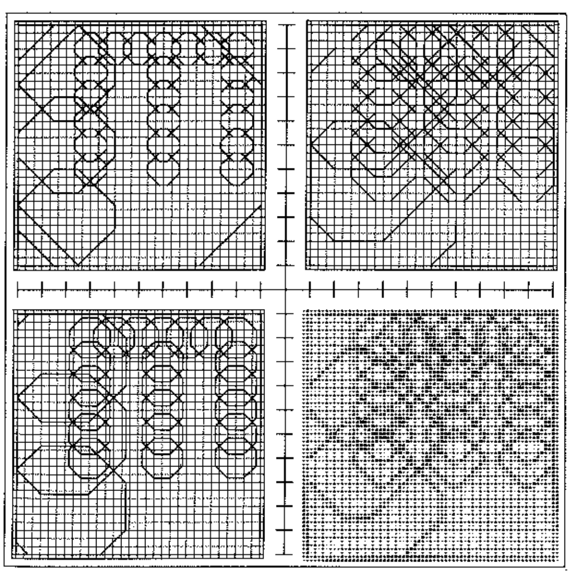
25. Pictures 28-31

The pictures 28;1-28;4, 29;1-29;4, 30;1-30;4, and 31;1-31;4 are line designs for inner-pictures. They present a square part of the plane of sP3 which is somewhat larger than the central cell of layer 0, more exactly: whose corner points are the points ((5/9);(5/9)), (-(5/9);(5/9)), (-(5/9);-(5/9)0, and ((5/9);-(5/9)). Moreover they present (for the interior of this square): 1 base-figure of layer 0 (and that the central one), 3 base-figures of layer 1 (on the left side of the picture), and 22 base-figures of layer 2 (they form a "M").

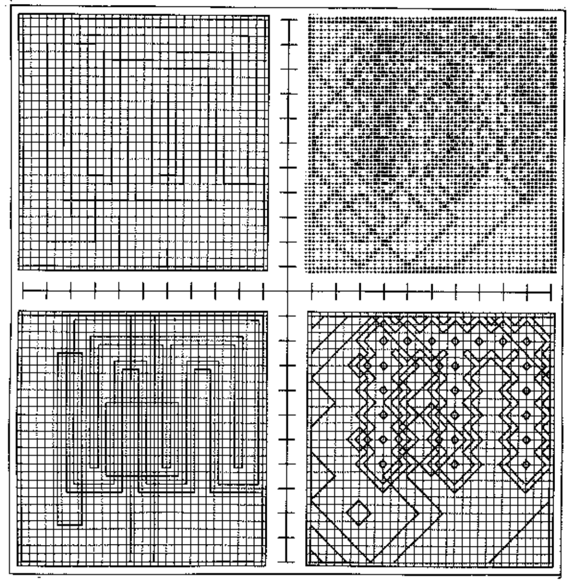
The pictures 28;1-28;4 are overlapping-pictures. The border-segments of their base-figures have the gradients ∞, 0, 1 and -1. Picture 28;2 presents the inner-star of inner-pictures representing objects published in the articles in this magazine in Fall 1978 and in Winter 1980. The pictures 29;1-29;4 are "fusion-pictures". The border-segments of their base-figures have the gradients ∞, 0, 1, and -1.

The inner-stars of the pictures 30;1-30;4, 31;1 and 31;3 have already been discussed. The pictures 31;2 and 31;4 are "fusion-pictures". They are co-ordinated to the inner-stars of the pictures 31;2 and 31;4 are "fusion-pictures". They are co-ordinated to the inner-stars of the pictures 9;4 and 9;5, 9;8 and 9;9, 9;12 and 9;13 respectively 8;(-3) and 8;(-2), 8;2 and 8;3, 8;6 and 8;7. The choice of the inner-stars, that underlies the design of the picture 31;2 and the one that underlies the design of the picture 31;4 can obviously be varied in nearly unserveyable ways.

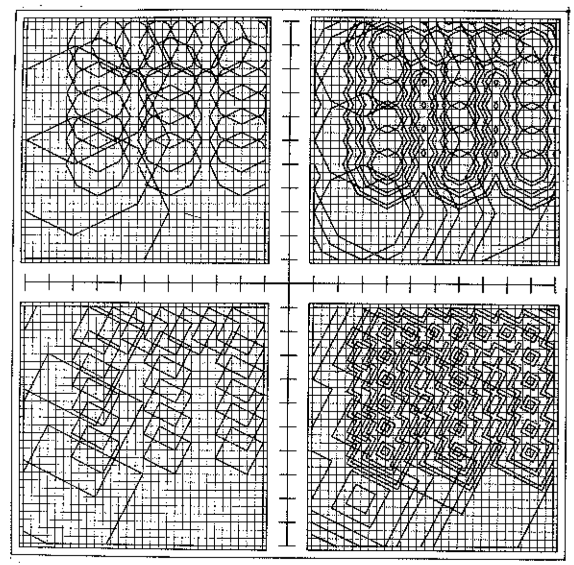
Picture 28.



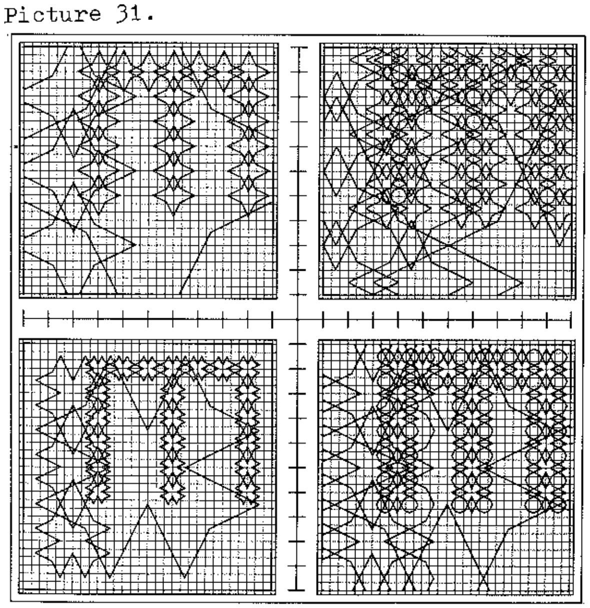
Picture 29.

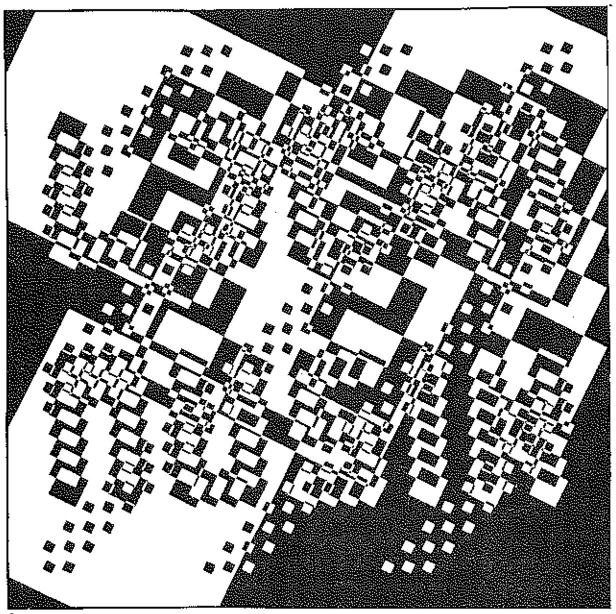


Picture 30.

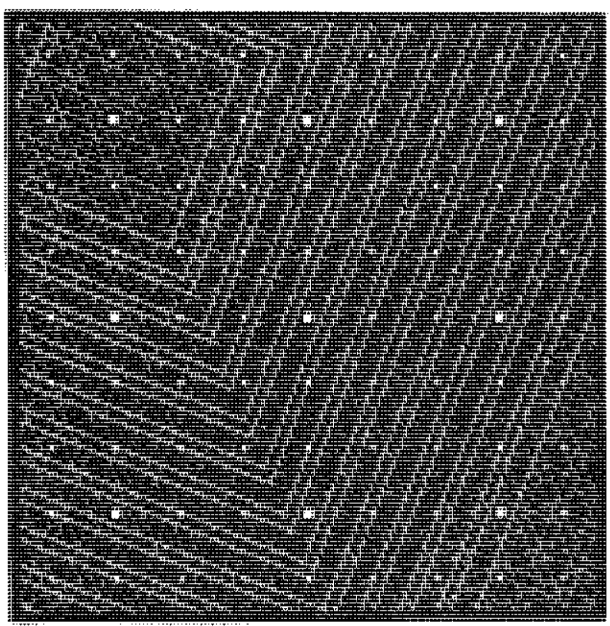


Picture 31.





Picture 32.



Picture 33.

26. Picture 32.

The picture 32 is an inner-picture of the inner-star of the pictures 8;3, 12, and 31;3. It presents the square part of the plane of sP3, already described in No. 25, and in it: 4 base-figures of layer 1, 22 of layer 2, forming a "M", 139 of layer 3, forming the stylized letters "LICHT-MUSIK", and 241 of layer 4 forming five ornamental rows.

27. Introduction of further gradients.

Once again we look at the pictures 8-11. The straight lines shown in them are co-ordinated to the sP3. They are strong- and field-lines of the gradients 2, -2, (1/2), and -(1/2). The field-lines belong to the number of division f=2. These strong- and field-lines form the layer 2 of a "complete-star". We call it the Compl(sP3; g=2, -2, (1/2), -(1/2); f=2). Furthermore the pictures 8-11 show numerous base-figures of layer 1 designed on this complete-star. Each of these Base-figures produces (at least) one inner-star of sP3.

Accordingly we define the complete-stars Compl(sP3; g=3, -3, (1/3), -(1/3); f=2), Compl(sP3; g-(3/2), -(3/2), (2/3), -(2/3); f=2), Compl(sP3; g=4, -4, (1/4), -(1/4); f=2), Compl(sP3; g=(4/3), -(4/3), (3/4), -(3/4); f=2), Compl(sP3; g=5, -5, (1/5), -(1/5); f=2). Compl(sP3; g=(5/2), -(5/2), (2/5), -(2/5); f=2) etc. In accordance with our procedure in pictures 8-11, we now design base-figures of layer 1 for the new complete-stars and therewith the appertaining inner-stars. The more "complicated" the used gradients are, the more numerous are the inner-stars to be obtained.

As demonstrated in picture 8 the gradients 2, -2, (1/2), -(1/2) (including those reflected on the axis of y) result in 6 inner-stars constructed with strong-segments, and 6 constructed with field-segments, of the kind constructed in picture 8. Thus, the gradients 3, -3, (1/3), -(1/3) result in 14 inner-stars constructed with strong-segments, and 14 constructed with field-segments, of the kind constructed in picture 8. Thus the gradients (3/2), -(3/2), (2/3), -(2/3) result in 20, and 20 such inner-stars already, the gradients 4, -4, (1/4), -(1/4) result in 26 and 26, the gradients (4/3), -(4/3), (3/4), -(3/4) in 42 and 42, the gradients 5, -5, (1/5), -(1/5) in 42 and 42, the gradients (5/2), -(5/20. (2/5), -(2/5) in 48 and 48, the gradients (5/3), -(5/3), (3/5), -(3/5) in 58 and 58, the gradients (5/4), -(5/4), (4/5), -(4/5) in 72 and 72.

28. Further methods for finding new "inner-stars".

The introduction of further gradients is the first and most important method for finding further "complete-stars" and thus inner-stars, too. A second method is the introduction of further numbers of division f for field-lines. In the case of sP3 the following numbers of division are possible: f = 2, 4, 5, 7, 8, 10, 11, 13, 14, etc. A third and last method is the following procedure: We compose the border lines of the base-figures of layer n not with strong- and field-segments of layer (n+1), but in addition or exclusively with strong- and field-segments of the layers (n+2), (n+3), ...

All classes of inner-stars, constructed by analogy with the class shown in picture 8 also allow the construction (among others) of "fusion-pictures" in the manner shown in picture 31;4. The same applies of course for the pictures 9 and 31;2. We can also design innumerable inner-stars of the "complete-star" of the pictures 8-11. --- Of course, innumerable pictures are possible representing intermediate forms between inner-pictures and other forms. The same applies for the inner-games.

29. Picture 33.

For the gradients (5/2), -(5/2), (2/5), -(2/5) picture 33 achieves the same as picture 8 for the gradients 2, -2, (1/2), -(1/2), however only for the base-figures whose border-lines are composed of strong-segments. The base-figure 33;1 is the smallest possible of its kind (composed of strong-segments). The base-figure 33;24 is the largest possible of its kind (composed of strong-segments) whose inner-star still has the attribute E.

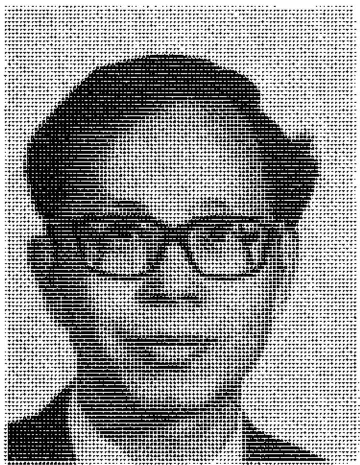
30. The "form-bases" of music and of "visual music".

An elementary sign of music is a resounding of a tone. A tone can resound: 1. in a definite pitch; 2. with a definite timbre; 3. with a definite sound intensity; 4. during a definite term --- altogether in a four-dimensional "space of attributes". Let us look at these four dimensions more closely: concerning 1.: The pitch can be chosen on the intervals of a scale, as a rule on the intervals of the scale of 12 tones; concerning 2.: The timbre can be chosen by selection from the musical instruments and the singing-voices at our disposal; concerning 3.: The sound intensity can be chosen theoretically from a one-dimensional continuity but practically only from a discrete multitude, viz. from finitely many degrees that can just still be distinguished; concerning 4.: The same applies for the term. At best about 10 tones can resound and be heard separately at the same time.

An elementary sign of the "dynamic visual music" is a flashing up of a base-figure in a certain inner-star. A base figure can flash up: 1. in a definite position above-below; 2. in a definite position right-left; 3. in a definite layer (which determines the size of the base figure, possible also its position forward-backward); 4. in a definite quality of color; 5. in a definite demness of color (white admixture); 6. in a definite light intensity; 7. during a definite term --- in all thus in a seven-dimensional "space of attributes". A closer study of these dimensions is left to the reader's discretion. In an inner-picture of sP3 at best all the base-figures of the layers 0-4, consequently 1+9+81+729 +6561 = 7381 base-figures, can appear at the same time, and be seen separately.

We call the system of all possible elementary signs of a work of music respectively of a work of the "visual music" its "form-base". The above statements and an exacter examination of the mentioned dimensions result in the statement: The "dynamic visual music" surpasses music essentially in regard of the number and extension of its possible from-bases.

AN ANALYSIS OF SCALES IN PICTORIAL DRAWINGS CONSTRUCTED BY HOHENBERG'S METHOD



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A new method of pictorial drawing called "Hohenberg's method" is emerging. This method combines both the old and the new: the century-old theory of orthographic projection and the new technique of pictorial projection by reflection. Instead of constructing pictorial drawings with the aid of specially prepared scales, a pictorial drawing is obtained directly by projection from two orthographic views by means of reflection. This method was developed by Fritz Hohenberg (1974) of the Austrian Technical University in Graz. Blade (1978) presented a paper on the basic concept of Hohenberg's method at the International Conference on Descriptive Geometry.

The purpose of this article is to present an analysis of scales in pictorial drawings constructed by Hohenberg's method, and, for logical purpose, to provide a brief review of projection procedures, advantages, and limitations underlying this method.

PICTORIAL DRAWINGS BY HOHENBERG'S METHOD

The key to Hohenberg's method of pictorial drawing is finding the point of reflection for each of the points from two orthographic views. Two principal views (such as front and top views) of a point in space are given as shown in Fig. 1. Move the point P_2 in the top view to anywhere on the projection plane as shown in P_r. Draw a line through P_r and P_1, and locate point P_f on the line so that $P_rP_f = K \times P_rP_1$ where K is any integer or integer fraction (K = 2 is most often used). Point P_f

is a simple reflection of P_r about P_1. Repeating this procedure to obtain the reflected point for each of the points from two orthographic views of an object, we can construct a paraline pictorial picture easily by connecting the reflected points in their pictorial order.

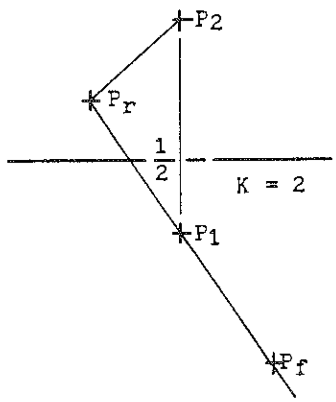


Fig. 1

PROJECTION PROCEDURES

To understand this method, let's take the example of a 1-unit cube to construct an axonometric drawing as shown in Fig. 2. The following procedures are used:

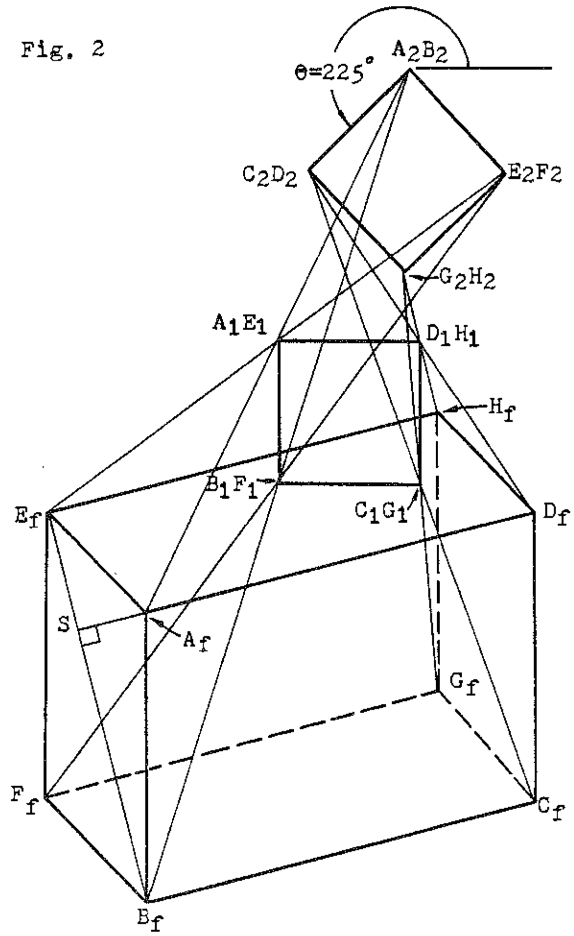
1. Draw front and top view of the cube.
2. Label each corresponding point properly in both views.
3. Rotate the top view anywhere on the projection plane. In this example, the top view is rotated 225° counterclockwise.
4. Connect points A_2 and A_1, and find the reflected point A_f of A_2, using K=2.
5. Repeat the procedure to find reflected points B_f, C_f, D_f, E_f, F_f, G_f, and H_f.
6. Connect the refected points in their pictorial order and we obtain an axonmetric affine view of the cube.

ANALYSIS OF SCALES

It can be seen from Fig. 2 that the scale ratios of the three axonometric axes of the axonometric affine projection obtained by Hohenberg's method are not congruent with the scale ratios obtained by the conventional methods of axonometric projection. However, certain important attributes of orthographic projection are conserved in Hohenberg's axonometric affine views as explained in the following:

1. All parallel lines of the cube are also projected parallel in the axonometric affine view.

Fig. 2



2. Since $\Delta A_2\text{-}A_1E_1\text{-}E_2$ and $\Delta A_f\text{-}A_1E_1\text{-}E_f$ are congruent, the reflected depth axis A_fE_f is equal to the length of A_2E_2, and parallel to, and opposite in direction of, A_2E_2. See Fig. 2 and Fig. 3. It can be concluded that if a line appears as a point view in the front view (the depth axis), its reflection will have the same length as shown in the top view ($A_fE_f = A_2E_2 = 1$ unit).

3. Since $\Delta A_2B_2\text{-}A_1\text{-}B_1$ and $\Delta A_2B_2\text{-}A_f\text{-}B_f$ are congruent, and line $A_2B_2\text{-}A_f$ is twice as long as line $A_2B_2\text{-}A_1$, the reflected height axis A_fB_f is twice as long as A_1B_1, and in

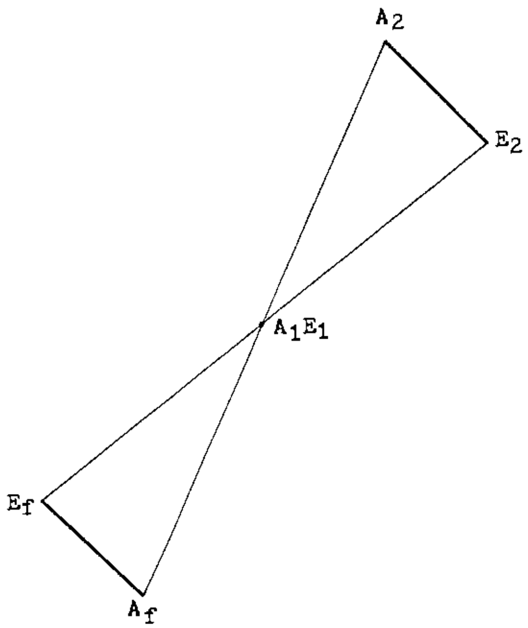


Fig. 3

the same direction. See Fig. 2 and Fig. 4. We can conclude that if a line appears as a point view in the top view (the height axis), its reflection will be twice the length and in the same direction as shown in the front view ($A_fB_f = 2\ A_1B_1 = 2$ units).

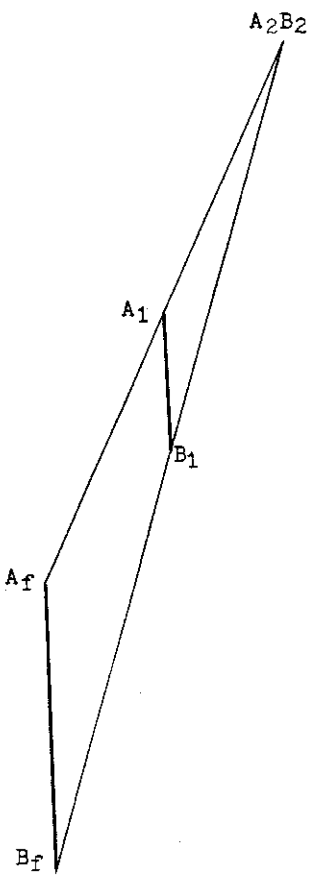


Fig. 4

4. The length and direction of reflected width axis A_fD_f are varied and are dependent on the angle of rotation of the top view. The length of this axis A_fD_f in units for K = 2 is

$$A_fD_f = \sqrt{5 - 4\cos\theta} \quad . . . . (1)$$

where θ is the angle through which the top view rotates counterclockwise.

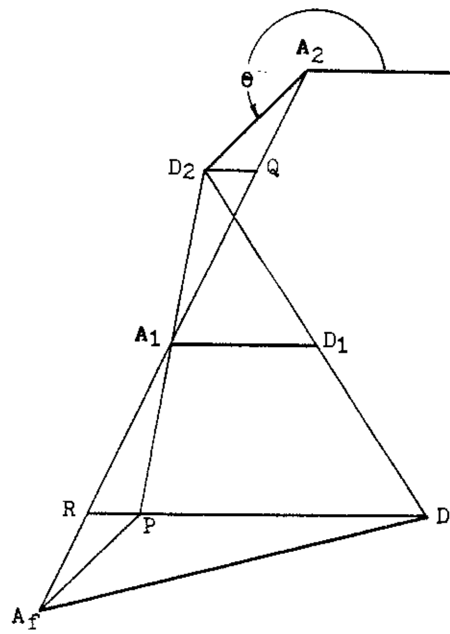


Fig. 5

To prove Eq. (1), referring to Fig. 5, draw line D_2Q parallel to A_1D_1, and line D_fR parallel to A_1D_1. Connect line D_2A_1 and extend to intersect line D_fR at P.

Draw line A_fP. Since $\Delta A_1D_2D_1$ and ΔPD_2D_f have three congruent angles, they are similar triangles and their sides are proportional. Thus,

$$D_2D_f = 2\ D_2D_1$$
$$P\ D_f = 2\ A_1D_1$$

and

$$A_1D_1 = 1 \text{ unit}$$
$$P\ D_f = 2 \text{ units}$$

Then in ΔA_1D_2Q and ΔA_1PR, we have

$$\angle D_2A_1Q = \angle PA_1R$$
$$\angle D_2QA_1 = \angle PRA_1$$

and

$$D_2A_1 = A_1P$$

Therefore,

$$\triangle A_1D_2Q \cong \triangle A_1PR$$

and $\quad RP = D_2Q$ . . . . ①

In $\triangle D_2QA_2$ and $\triangle PRA_f$, we have

$\angle A_fRP = \angle A_2QD_2$ . . . . ②

Since $A_2A_1 = A_1A_f$, and $A_1Q = A_1R$, we obtain

$A_fR = A_2Q$ . . . . ③

From conditions ①, ②, and ③, we prove

$$\triangle D_2QA_2 \cong \triangle PRA_f$$

and $\quad PA_f = D_2A_2$

thus, $\quad PA_f = 1$ unit

Also in $\triangle A_fPD_f$, we know $A_fP \parallel D_2A_2$,

then $\quad \angle A_fPD_f = 360^\circ - \theta$

Thus, in $\triangle A_fPD_f$, according to the cosine law, the length of A_fD_f is

$$A_fD_f = \sqrt{(PD_f)^2 + (PA_f)^2 - 2(PD_f)(PA_f)\cos \angle A_fPD_f}$$
$$= \sqrt{2^2 + 1^2 - 2(2)(1)\cos(360^\circ - \theta)}$$
$$= \sqrt{5 - 4\cos\theta} \quad \text{units}$$

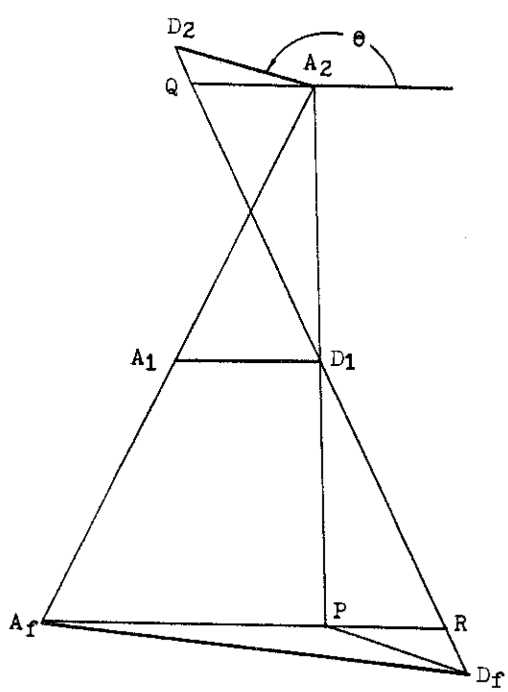


Fig. 6

In Fig. 6 where the angle of rotation θ is $180^\circ > \theta > 90^\circ$, and in Fig. 7 where θ is $90^\circ > \theta > 0^\circ$, we also obtain

$A_fP = 2$ units

$D_fP = 1$ unit

and $\quad \angle A_fPD_f = \theta$.

thus prove

$$A_fD_f = \sqrt{2^2 + 1^2 - 2(2)(1)\cos\theta}$$
$$= \sqrt{5 - 4\cos\theta} \quad \text{units}$$

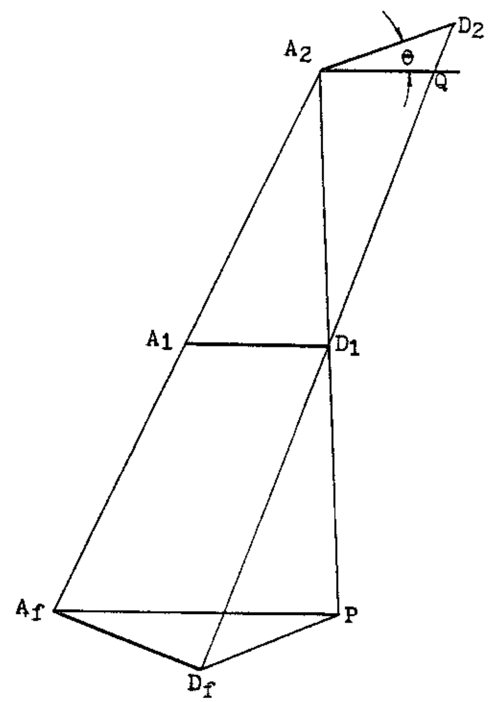


Fig. 7

As an example, the angle of counter-clockwise rotation shown in Fig. 2 is 225°, and hence, the length of A_fD_f is:

$$A_fD_f = \sqrt{5 - 4\cos 225^\circ}$$
$$= \sqrt{5 - 4(-0.7071)}$$
$$= \sqrt{5 + 2.8284}$$
$$= 2.7979 \text{ units}$$

Since the values for cos θ range from 0 to $\pm$ 1, the length of the width axis will fall within the range of 3 units and 1 unit. The width axis will be at its longest (3 units) when the rotated angle is 270°, and at is shortest (1 unit) when rotated 0°.

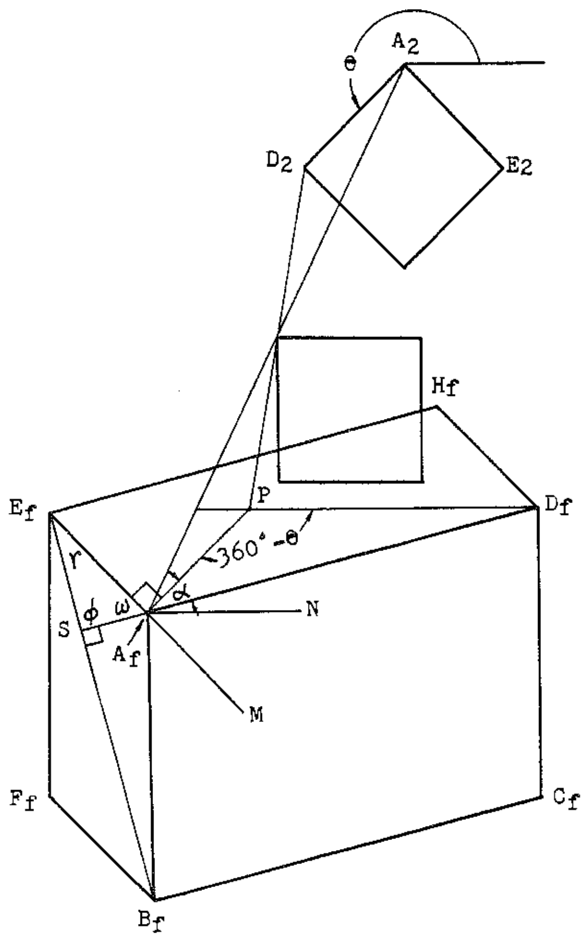


Fig. 8

The direction of the width axis A_fD_f is always perpendicular to the diagonal connecting the ends of the depth axis and the height axis. In other words, A_fD_f is perpendicular to line B_fE_f. To prove the perpendicularity, refer to Figures 2 and 8. In Fig. 8, draw a horizontal line A_fN, and extend E_fA_f to M. Since $E_fA_f /\!/ A_2E_2$, and A_fB_f is a vertical line, we have

$$\angle MA_fN = 360^{\circ} - \theta - 90^{\circ} = 270^{\circ} - \theta$$

$$\angle B_fA_fM = 90^{\circ} - \angle MA_fN = \theta - 180^{\circ}$$
$$(270^{\circ} > \theta > 180^{\circ})$$

and

$$B_fA_fE_f = 180^{\circ} - \angle B_fA_fM = 360^{\circ} - \theta$$

since $\angle A_fPD_f = 360^{\circ} - \theta$

thus $\angle A_fPD_f = \angle B_fA_fE_f$

also $PD_f = A_fB_f = 2$ units

$A_fP = A_fE_f = 1$ unit

thus $\Delta A_fPD_f \cong \Delta E_fA_fB_f$

and $\angle \alpha = \angle \gamma$ . . . . ①

Since

$A_fP /\!/ A_2D_2$

$\angle PA_fE_f = 90^{\circ}$

$\angle \omega + \angle PA_fE_f + \angle \alpha = 180^{\circ}$

$\angle \omega + 90^{\circ} + \angle \alpha = 180^{\circ}$

$\angle \omega + \angle \alpha = 90^{\circ}$. . . ②

In ΔA_fSE_f, we know $\angle\omega + \angle\gamma + \angle\phi = 180^{\circ}$

from ①, $\angle \omega + \angle \alpha + \angle \phi = 180^{\circ}$

from ②, $90^{\circ} + \angle \phi = 180^{\circ}$

thus $\angle \phi = 90^{\circ}$

Therefore, we prove $A_fS \perp B_fE_f$

and $A_fD_f \perp B_fE_f$

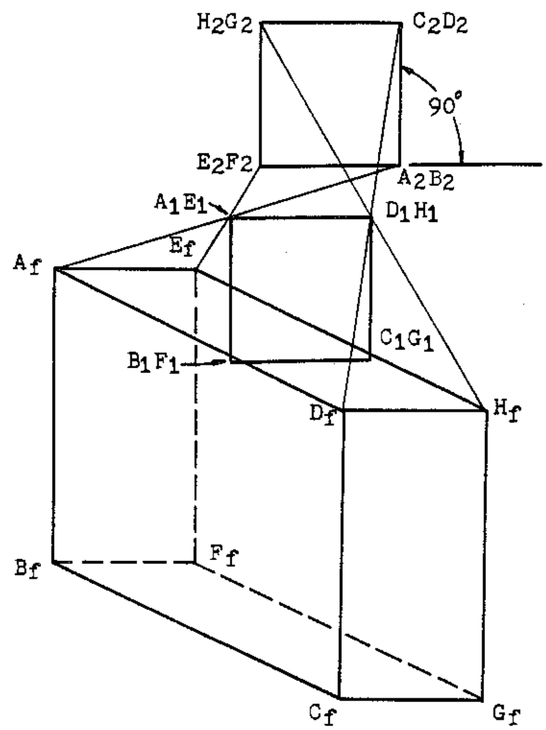


Fig. 9

5. When the top view is rotated counterclockwise 90°, the pictorial view by reflection with $K = 2$ appears like an oblique drawing as shown in Fig. 9. The profile surface $D_fC_fG_fH_f$ becomes a rectangle with $D_fH_f = D_2H_2$ and $D_fC_f = 2D_1C_1$.

The length of A_fD_f is:

$$\begin{aligned} A_fD_f &= \sqrt{5 - 4 \cos 90^{\circ}} \\ &= \sqrt{5 - 0} \\ &= 2.2361 \text{ units} \end{aligned}$$

6. The combination of front and right-side views can also be used with the right-side view rotating to draw paraline pictorial pictures. The procedures are essentially the same.

CRITIQUE

Hohenberg's method of pictorial drawing through rotation and reflection has certain advantages and limitations. Among the advantages are:

1. Important features of orthographic projection and the properties of affine geometry are preserved. All parallel lines shown on multiview drawings remain parallel in the pictorial views. Scale ratios for the width, height, and depth axes are maintained.

2. Since the top view (or side view) can be rotated any angle, this method has the advantage of showing an object in many views. The pictorial views can be in any form of axonometric or oblique drawing.

3. It may have the advantage of simplicity in making certain pictorial drawings of objects with irregular configurations as pictorial drawings are projected directly from two orthographic views. See Fig. 10.

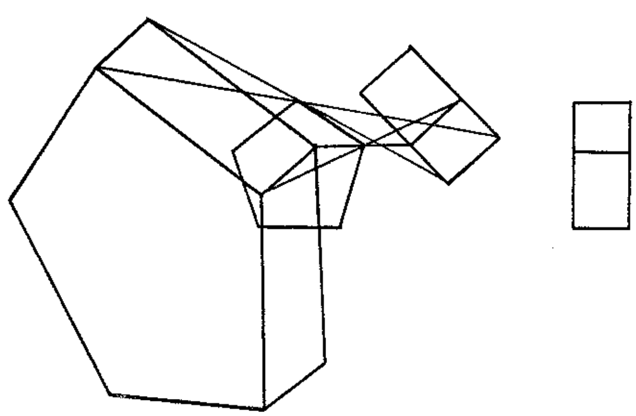


Fig. 10

The limitations of Hohenberg's method for constructing pictorial drawing can be summarized as follows:

1. This method requires the identification of each corresponding point in the multiview drawings, which may be tedious, difficult, and time-consuming, especially for more complicated objects.

2. The pictorial drawings obtained from this method with certain angles of rotation are extremely distorted such as the one shown in Fig. 11.

3. It is very difficult to draw objects that have circular or curved lines. Existing ellipse templates and other drawing aids are not available for this type of drawing.

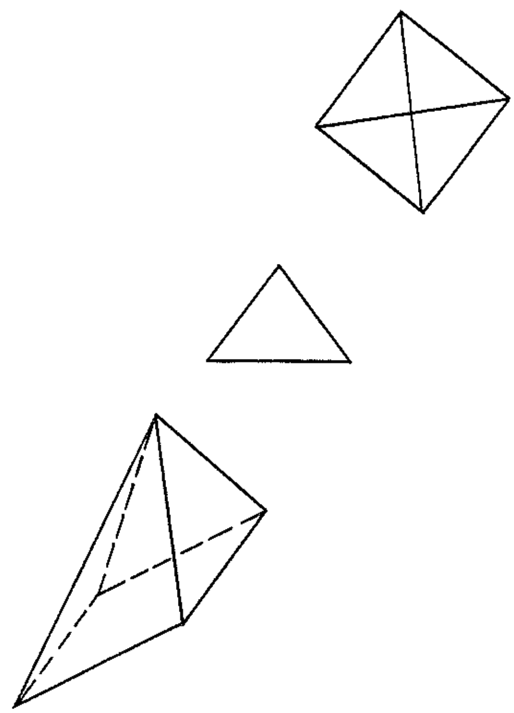


Fig. 11

SUMMARY

In sum, Hohenberg's method of paraline pictorial drawing offers a viable alternative to pictorial drawing. It is relatively simple to obtain a pictorial view of an object by means of reflection from two multiview drawings. However, this method is not without certain limitations. Due to its brief history, Hohenberg's method of paraline pictorial drawing is still in a very preliminary stage. More researches are needed to further explore its theory and applications so that this method can become a useful means for making pictorial drawings.

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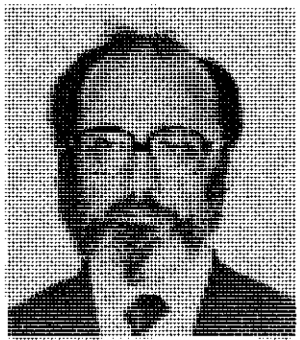
ON CONDITIONAL INVARIANTS OF ORTHOGRAPHIC PROJECTIONS



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1. DEFINITIONS AND SCOPE

An invariant of a geometrical transformation is a property of a geometrical configuration which remains unchanged under this transformation. For example, the ratio AB/CD of two lines AB// CD is an invariant of the parallel-projection transformation since, in any parallel projection which transforms two lines AB and CD into two lines ab and cd, AB/CD = ab/cd.

We define a conditional invariant of an orthographic projection as a property of a geometrical configuration which remains unchanged if and only if one or more specific conditions relating the direction of viewing to this configuration is satisfied. For example, perpendicularity of two lines is a conditional invariant of an orthographic projection since two perpendicular lines appear as perpendicular in an orthographic projection if and only if the condition is satisfied that at least one of the two lines is perpendicular to the direction of viewing.

This paper examines the following two conditional invariants of orthographic projections:

a. Constancy of an angle other than $\pi/2$ between two intersecting lines, and

b. Constancy of a ratio of two arbitrary lines.

2. CONSTANCY OF AN ANGLE

Theorem:

Let AB and CD (Figure 1)* be two non-perpendicular lines intersecting at the point O ($\angle AOC < \pi/2$) and M, a point on CD such that AM $\perp$ CD; then constancy of

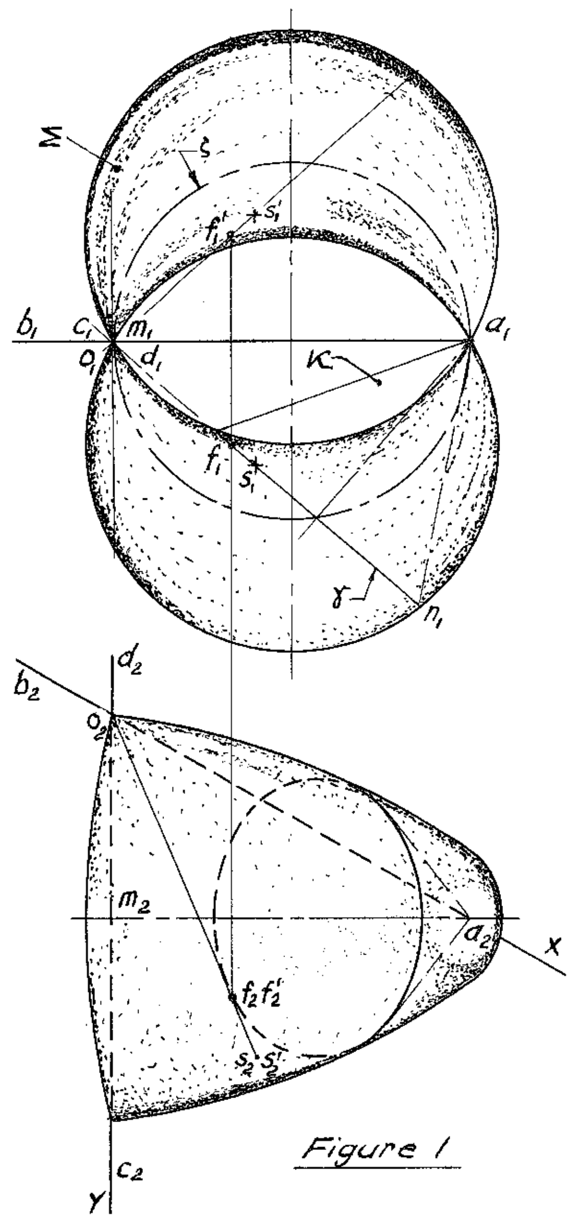


Figure 1

*) In this paper, lower case letters are used to denote the projections of the respective points in space. In two-view orthographic representations, subscripts 1 and 2 are used.

of the angle between the lines AB and CD is a conditional invariant of orthographic projection if and only if the direction of viewing is parallel to any line other than OC but containing the point O and tangent to the surface which is a geometric locus of all circles γ such that

(i) γ is in a plane containing CD;
(ii) the centre of γ is a point of a circle ξ with AM as its diameter and in a plane normal to CD;
(iii) the diameter of γ varies so that all cones ζ with apex A and directrices γ have included apex angle equal to $\pi - 2.\ \angle AOC$.

Proof:

Consider any one conical surface ζ (Figure 2) with its apex at the point A and directrix γ . Let P be the plane containing γ and E the centre of γ .

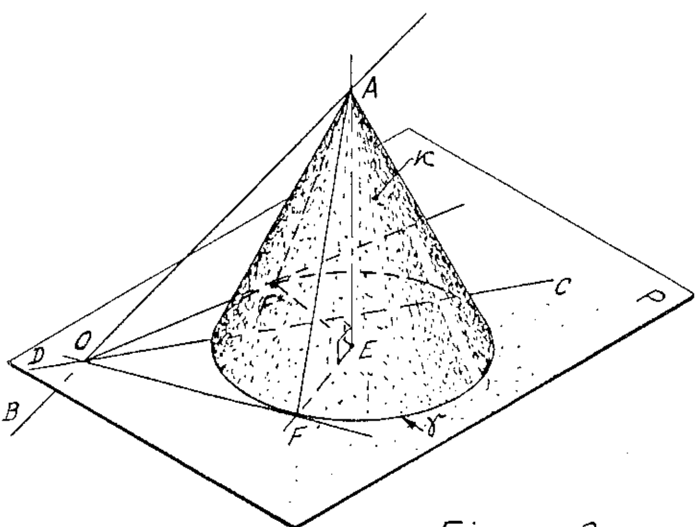


Figure 2

For any point O in the plane P outside the circle γ there exist two tangent lines OF and OF* to γ . Since γ belongs to the surface Σ , OF and OF* are also tangent to Σ . Since OF ⊥ EF and OF* ⊥ EF*, OF ⊥ AF and OF* ⊥ AF* (cf. the "theorem of the three perpendiculars": if a line OF in a projection plane P is perpendicular to the orthographic projection EF of a line AF on the plane P, then OF ⊥ AF). It follows that OF and OF* are perpendicular to the planes containing the angles AFE and AF*E respectively and, depending on the position of OC relative to γ (v. Note "b" below), each of the angles AFE and AF*E is an orthographic projection of $\angle COA$ or $\angle DOA$. But

$$\angle AFE = \angle AF^*E = \pi/2 - \angle FAE$$

$$= \angle COA.$$

Thus, we have shown that the angle between the lines AB and CD remains unchanged in the orthographic projections in the directions of OF and OF*.

To prove the converse part of the theorem, assume that OF is a line not tangent to Σ . Then γ does not belong to Σ and the assumption leads to a conclusion that there exists a circle γ satisfying conditions (i), (ii), and (iii) and not belonging to Σ . Such conclusion contradicts the definition of Σ as a geometric locus of all circles γ .

Thus, the theorem is proved.

NOTES:

a. The locus of all the lines OF and OF* tangent to Σ is a conical surface Λ with its apex at the point O;

b. If OC (Figure 2) does not intersect the circle γ , one of the two angles, $\angle AFE$ or $\angle AF^*E$, is an orthographic projection of $\angle AOC$ while the other is an orthographic projection of $\angle AOD = \pi - \angle AOC$;

c. Since, in Figure 1, $\angle c_1n_1a_1 = \angle c_2o_2a_2$ = constant, the locus of points n_1, i.e. the outline of Σ in the view along CD, consists of two identical circles with the common chord o_1a_1.

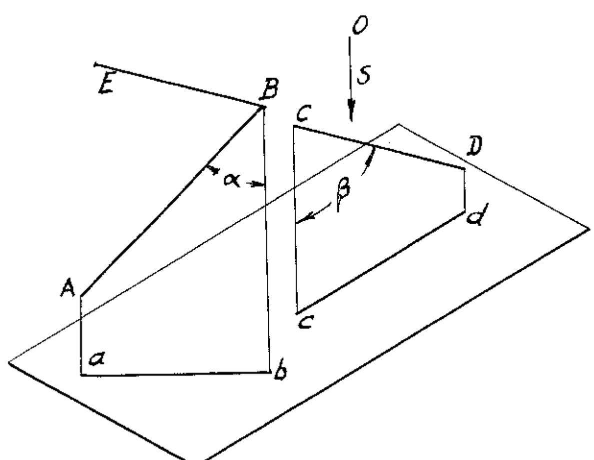


Figure 3

3. CONSTANCY OF A RATIO OF TWO ARBITRARY STRAIGHT LINES.

Let AB and CD (Figure 3) be two non-parallel lines and ab and cd their respective orthographic projections in the direction OS forming angles α and β ($\alpha \leqslant \pi/2$ and $\beta \leqslant \pi/2$) with AB and CD respectively. Then,

$$ab = AB \sin \alpha$$

and

$$cd = CD \sin \beta$$

and, hence,

$$\frac{AB}{CD} = \frac{ab \sin\beta}{cd \sin\alpha}$$

It follows that a necessary and sufficient condition for AB/CD = ab/cd is that $\alpha = \beta$

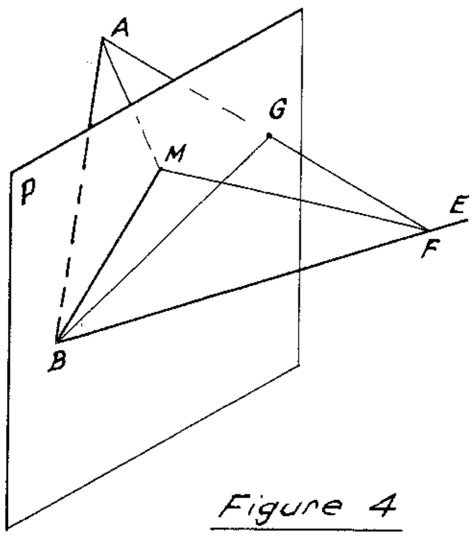


Figure 4

Let now E be a point such that BE // CD and BE = CD. We shall show that the geometrical locus of all directions OS of orthographic viewing satisfying the condition $\alpha = \beta$ is a plane P (Figure 4) normal to the plane ABE and containing the bisector BG of the angle ABE. Let F be a point on BE such that FB = AB; then the plane P is the geometrical locus of all points M equidistant from the points A and F. It follows that Δ AMB = Δ FMB and, hence $\angle$ ABM = $\angle$ EBM. Thus, in any orthographic view in the direction BM, AB/BE = ab/be. But, since BE// CD and BE = CD,

$$AB/CD = ab/cd.$$

Also, it can be seen that, for any point M not belonging to the plane P, AM $\neq$ FM and $\angle$ ABM $\neq$ $\angle$ EBM and, hence, AB / CD $\neq$ ab / cd. Thus, we have shown that constancy of a ratio AB/CD of two non-parallel lines AB and CD is a conditional invariant of orthographic projection if and only if the direction of viewing is parallel to the plane P $\perp$ ABE (BE // CD, BE = CD) which contains the bisector of the angle ABE.

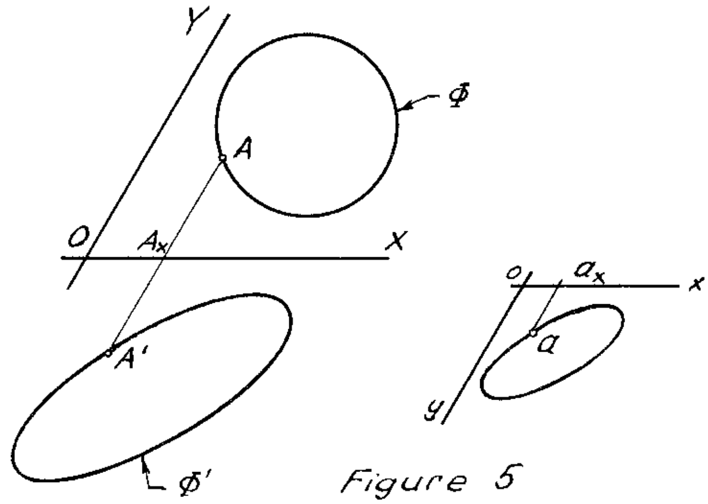


Figure 5

4. AN EXAMPLE OF APPLICATION : SKEW REFLECTION.

Let OX and OY (Figure 5) be two intersecting lines and Φ any line figure in the plane OXY; then the line figure Φ' is the skew reflection of Φ in OX parallel to OY if Φ' can be positioned so that to every point A of Φ there corresponds a point A' of Φ' and, conversely, to every point A' of Φ' there corresponds a point A of Φ such that AA' // OY and $\vec{A'A_x} = \vec{A_xA}$, where A_x is the point of intersection of AA' with OX. We shall show how the conditional invariants formulated in Sections 2 and 3 of this paper may be used to solve the following problem:

Given: two intersecting lines OX and OY and an arbitrary line figure Φ in the plane OXY.

Required: to construct a line OS such that the orthographic view of Φ in the direction OS is a scaled skew reflection of Φ in OX parallel to OY.

Strategy for the solution:

The required line OS must satisfy the conditions that, in the orthographic view in direction OS,

(i). $\angle xoy = \pi - \angle XOY$, and

(ii) $\frac{oa_x}{a_x a} = \frac{OA_x}{A_x A}$.

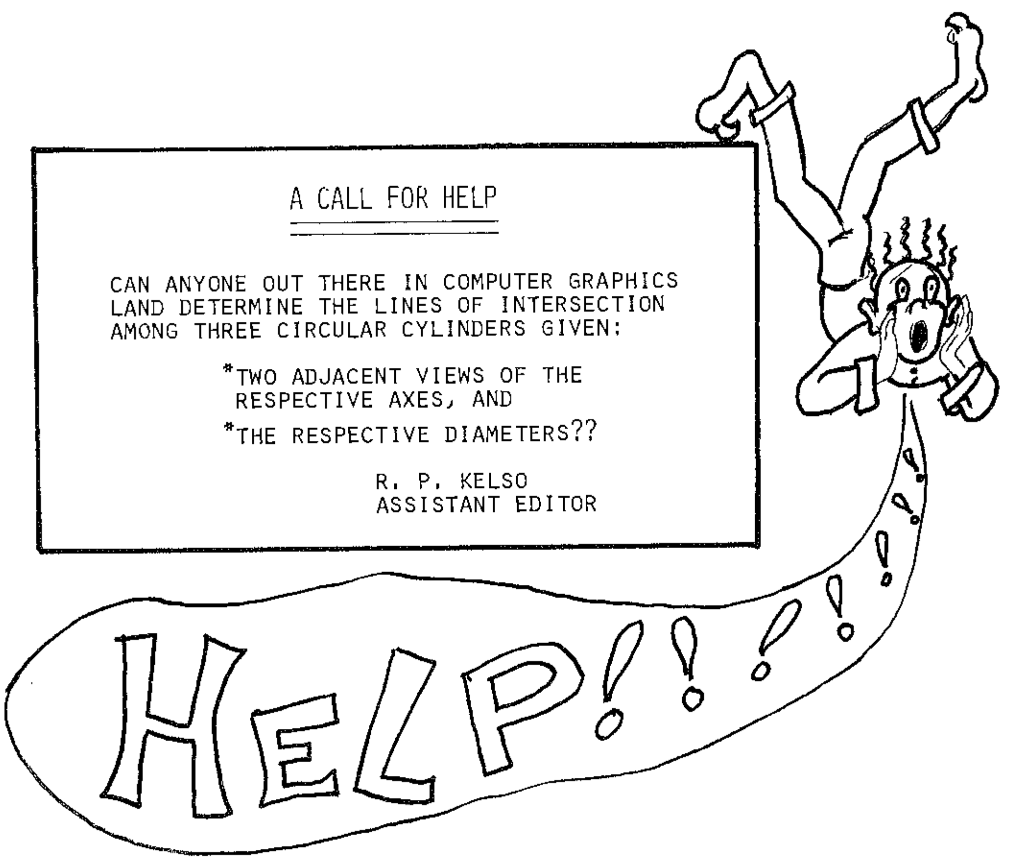
The locus of all directions of orthographic viewing satisfying condition (i) and the condition that ∠ xoy = ∠ XOY is the conical surface Λ described in Section 2 (v. Notes a and b). The locus of all directions of orthographic viewing satisfying condition (ii) is the plane P described in Section 3. Thus, OS may be found as one of the lines of intersection of P with Λ . For example, if in Figure 1 o_2f_2 is the bisector of the angle $a_2s_2c_2$, then OS and OS' are the solutions of the problem (OX and OY in Figure 1 are coincident with AB and CD respectively).

5. CONCLUDING REMARK.

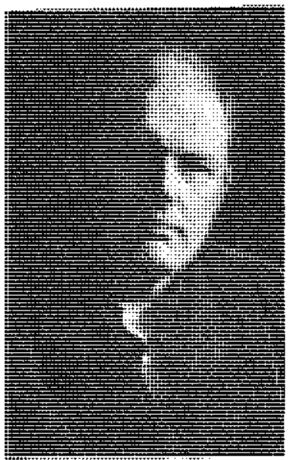
This paper deals with conditional invariants only, but the two theorems formulated in it may be extended to define conditions for any arbitrary angle between two lines to be represented by any other arbitrarily selected angle between their projections, and conditions for a ratio of two arbitrary lines to be represented by any other arbitrarily selected ratio of their projections. For example, if in Figure 1, the cone ζ has an arbitrary (but subject to certain conditions) apex angle $\pi - 2\psi$, the surface Λ is the locus of directions of orthographic viewing such that the given angle ∠ AOC appears as the given angle ψ (or $\pi - \psi$), ψ not necessarily being equal to ∠ AOC. A detailed study of this type of problem and its applications may prove to be very rewarding.

ACKNOWLEDGEMENT:

The author expresses his appreciation to the Editor of the EDGJ "Puzzle Corner", Pat Kelso, whose intriguing puzzles inspired the writing of this paper.



THE GRAPHIC PLOTTING OF COMPLEX SLOPES OF ORDINARY DIFFERENTIAL EQUATIONS



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INTRODUCTION

If the orthogonal projection of the four co-ordinates of a four dimensional Euclidean Cartesian graph, as projected onto a two-dimensional surface, is interpreted such that two of the coordinates represent real numbers and the remaining two coordinates represent imaginary numbers, then such a graph allows the representation of two-variable complex functions. Any such function is thus a surface in complex 4-space.

Any angular distance from the origin in this complex 4-space may be considered as real, imaginary or isotropic
as a real, imaginary, or isotropic (0) distance under a new definition of the absolute value of a complex number:

$$\{Z\} = \sqrt{a^2 + (bi)^2}$$

in which the Pythagorean Theorem holds for the complex plane and for complex 4-space.*

* Brisson, David W., "Complex Space/Time:An Alternative to the Special Theory of Relativity", MODELING AND SIMULATION, VOLUME 10, Proceedings of the Tenth Annual Pittsburgh Conference, University of Pittsburgh, Ed. William G. Vogt and Marlin H. Mickle, School of Engineering, Univ. of Pittsburgh, Instrument Society of America, Pittsburgh, 1979., ppgs. 1983-89.

Since the derivative of a differential equation is dependant upon the Pythagorean Theorem, it follows -- convention to the contrary -- that any ordinary differential equation has a whole class of complex slopes in addition to the conventional real slopes that form a surface in complex 4-space that is a graphic expression of the derivative.

To demonstrate how this may be done, the equation for the vertex parabola has been chosen and its derivative found. Values are then found considering both real and imaginary values of X. These values are then plotted in a complex four-dimensional graph such as described.

THE DERIVATIVE OF THE VERTEX PARABOLA

Taking the equation for the vertex parabola:

$$Y^2 = 2pX, \qquad (1)$$

where $\frac{p}{2}$ = the distance from the focus of the parabola to the vertex, and substituting X for Y, and vice versa, for purposes of simpler manipulation of the derivative, we arrive at the equation:

$$Y = \frac{X^2}{2p} \quad \text{or} \quad f(x) = \frac{x^2}{2p} \qquad (2)$$

Considering the general equation for the derivative:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \qquad (3)$$

$$f(x+\Delta x) - f(x) = \frac{x^2+2x\,\Delta x+(\Delta x)^2-x^2}{2p} \qquad (4)$$

$$= \frac{2x\,\Delta x + (\Delta x)^2}{2p}$$

Dividing by Δx we then obtain:

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{2x + \Delta x}{2p} \qquad (5)$$

Then:

$$f'(x) = \lim_{\Delta x \to 0} \frac{2x + \Delta x}{2p} \qquad (6)$$

and:

$$m = \frac{x}{p}\text{, the slope.} \qquad (7)$$

Substituting various real and imaginary values for x in equations (2) and (7) we arrive at the following table of values for p=1:

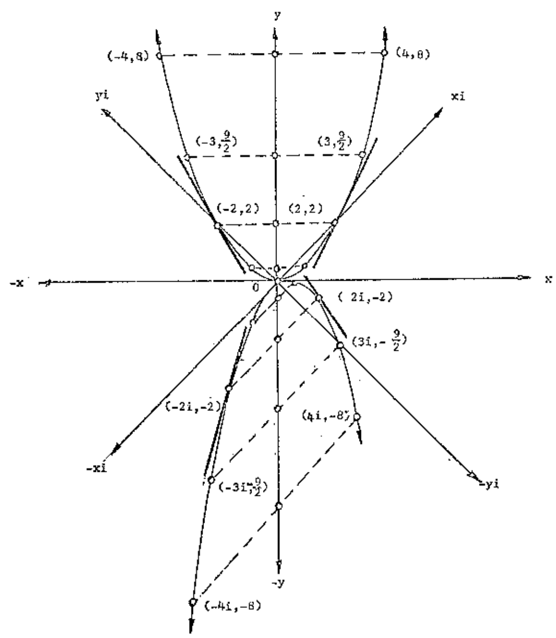


Figure 1.

x	-4	-3	-2	-1	0	1	2	3	4	-4i	-3i	-2i	-i	0	i etc.
y	8	$\frac{9}{2}$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2	$\frac{9}{2}$	8	-8	$-\frac{9}{2}$	-2	$-\frac{1}{2}$	0	$-\frac{1}{2}$
m	-4	-3	-2	-1	0	1	2	3	4	-4i	-3i	-2i	-i	0	i

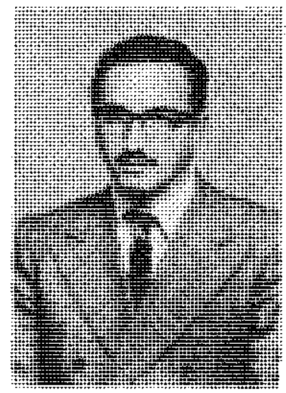
TABLE I

Plotting these values of x and y in a complex 4-space graph, we arrive at the curves in Figure 1. Note that these two curves in the real x-y plane and the complex yxi plane are cross-sections of a curved surface in complex 4-space described by equation (2).

The slopes at points (-2,2) and (2,2) have been sketched in and obviously conform to conventional practice. Considering now the point (-2i,-2), one must recognize that m is a ratio of y to x. Thus, m at (-2,2) is the ratio of y to one unit of x, or -2:1, or -2. Similarly, (-2i,-2) is a ratio of m; i.e., y to one unit of x, or -2i:-i = 2. If p had equaled i, for example, and x had equaled 2i, then y would equal $-\frac{2}{i}$, m would equal 2 and the ratio of m to xi would equal 2:i, i.e., the curve is purely imaginary and so is the tangent. Point (2i,-2) has also been sketched in.

Thus, if a derivative can be obtained for an equation, it may be expressed in complex terms.

BIRTH OF ENGINEERING REPROGRAPHICS



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BIRTH OF ENGINEERING REPROGRAPHICS

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Reproduction through light.

A revolutionary invention has placed on a new basis the reproduction technique. It was a question of reproducing drawings through the help of light, the so-called photocopying process. By photocopying we mean the production of copies of drawings penetrable by light, without the use of a camera.

The first photocopy was carried out in 1727 by the German physician Johann Heinrich Schulze. He found that a mixture of silver nitrate and calcium sulphate, if exposed to sunlight but not to heat, becomes black. He obtained the first copy by putting patterns of letters upon the light-sensitive mixture. However, there was still not a method to fix the image. Therefore the process in practice did not come into use. Yet the research into the sensitivity to light of silver salts was carried on. Many well-known researchers took part in this study, till the English Th. Wedgwood, in 1802, succeeded in producing shadows thanks to the effect of sunlight on paper or glass imbued with silver nitrate. He mostly made use of translucent (transparent) objects, like insects' wings, leaves and the like. Nevertheless he was still not able to fix the images.

In the year 1819 Sir John Herschel, astronomer and physicist (Fig 1), discovered the effect of sodium hyposulphite on silver salts. However, this discovery at first was not object of attention. Only in 1839 W. H. Fox Talbot (Fig. 2), following the indications given by Sir John Herschel, for his very sensitive and recently discovered silver chloride paper, used for the first time sodium hyposulphite to fix the images. First of all he used the silver chloride paper to copy drawings and manuscripts, therefore he was the first to use the method that later on will be called the photocopying process. He also knew the way to copy leaves and flowers. The first negatives on paper had white lines and a black background. From these it was possible to obtain positives too. In 1840, Talbot improved his method by spreading gallic acid on silver iodide paper, rendering it light-sensitive. By this method, Talbot succeeded in notably shortening the time of exposure to light. So Talbot must be considered the founder of photocopy and of the photocopying technique.

Fig. 1: Sir John Herschel,
born on March 7, 1792 at Slough (near Windsor),
died on May 2, 1871 at Collingwood.
Photo: Bildarchiv der Österr. Nationalbibliothek

Fig. 2: William Henry Talbot,
born on February 1800 at Harrow,
died on September 17, 1877 at Laycock Abbey.
Photo: Bildarchiv der Österr. Nationalbibliothek

However, a suitable material for the copies had still not been found. The various papers for copying had no resistance. They had to be used if possible on the same day on which they had been prepared. Besides the paper spread with silver emulsion was too expensive to be utilated for photocopies. This gap could be filled only when, in the eighties of the 19th century, people began to photocopy on paper treated with silver bromide gelatin. In the meanwhile in the photocopying technique a simple and less expensive process became known: the blueprint process.

The blueprint process (Negative cyanography).

Already in the year 1831 Dr. Wolfgang Döbereiner (1780-1849) observed the effect of light on iron oxalates. In 1840 Sir John Herschel found that generally the ferric salts of organic acids are reduced by light to ferrous salts. In 1842, he succeeded in obtaining the first copy. He prepared some paper with an aqueous solution at 10% of this ferric salt, threw light under a drawing, then plunged the copy into potassium fericyanide and so he obtained a white drawing on a blue background; the bluepring process or cyanography was born. For fixing it was sufficient to plunge the copy into pure water. This process is simpler because paper is embued with potassium ferricyanide and iron-ammonium citrate. Once anyone who made the photocopy had to prepare the paper by himself. Today it is produced by a few firms and it is supplied ready for use in standard sizes or in rolls. When it has dried, the paper has a yellow-green colour.

In order to bring about the illumination, the original tracing ais set with the back of the sheet on the sensitive layer, in a proper frame for photocopies, and it is exposed to sunlight, otherwise it is put in a photocopying device constituted by a glass cylinder and it is exposed to the light of an arc lamp or of a mercury-vapor lamp. After the photocopying process, the blueprint is washed in pure water. The copy has white lines on a blue background. If we want to obtain clearly readable blueprints the originals must be written in China ink. From a blueprint it is not possible to obtain another blueprint directly. Blueprints have a duration of more than 50 years. The blueprint process has been used from 1867 in England and in France, and in the same year it was displayed at the World Exhibition of Paris. This process, initially used by amateurs, as years went by became indispensable for the reproduction of technical drawings.

Positive cyanography.

The bases of this process were given by Herschel already in 1842. If some paper, prepared with iron-ammonium citrate, is illuminated under a drawing and then is treated with potassium ferrocyanide, one obtains a positive figure. In the year 1877 Pellet improved this process, spreading on the paper a rubber solution too. A further improvement was produced in 1880 by Collache. In the year 1881 the iron-rubber process was described for the first time by the Austrian captain Beppo Pizzighelli (Fig 3). Pizzighelli made some experiments and at last chose three solutions, that must be mixed in a given succession. Paper is spread with these solutions. The developing agent is constituted by potassium ferrocyanide. The drawing has blue lines on a white background. At last the copy is washed clean from the developing agent in pure water. The iron-rubber paper is very sensitive to light and photocopying on it happens like on the blueprint paper.

The sepia reproduction process.

The blueprint process with iron was also combined with the silver salts process, for the first time in 1844 by Hunt and in 1857 by Draper. The paper required for this process was put on the market by Nicol in the year 1889 with the name Kallityppapier (paper of the type kalli). On the grounds of these preliminary works, Arndt and Troost (1894) obtained the so-called paper for sepia photocopies. It is produced spreading a gelatinous solution of ammonium-iron citrate, silver nitrate and tartaric acid. Sepia paper is illuminated under a transparent drawing, until the lines begin to colour and the borders show a dark brown colouring. It is then washed with water and fixed in a soda solution at two per cent. After fixing one must wash well. One obtains white lines on a brown background (negative copies). These, if they are photocopied on light paper, can be

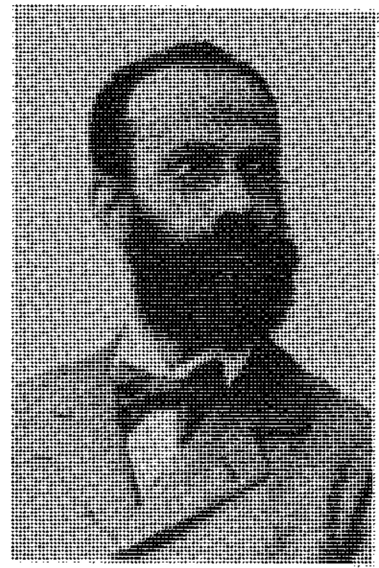

Fig. 3: Beppo Pizzighelli, born on December 28, 1849 in Mantua, died on April 15, 1912 in Florence. Photo: Bildarchiv der Österr. Nationalbibliothek

utilized in order to produce positive copies on sepia-type or blueprint paper. The sepia photocopying process was utilized until about 1930. It then replaced, like cyanography too, by the new diazotype process.

The diazotype process.

In the year 1860 the german chemist Peter Griess produced for the first time diazo compounds. Afterwards in 1900 various photocopying processes were developed, on the grounds of the light-sensitivity of diazo compounds. Most of these processes, however, have found no practical use. Only the process of Prof. Gustav Kögel, born in Munich in January 16th 1882, succeeded in being successful as a photocopying technique.

The Ozalid photocopying process.

The story of ozalid photocopies began at Beuron, near Munich. There Professor G. Kögel was busy deciphering the palimsests, viz. handwritten documents on parchment, that had been scratched out (in Greek: palim psestòs) and written on again. In order to be able to read the erased pristine writing,

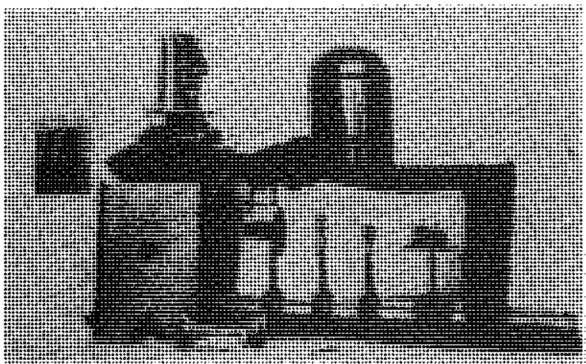

Fig. 4: Device of G. Kögel for the decomposition of the rays, for the palimpsest-photography.

he invented by himself, for the decomposition of the rays (Fig. 4), by which he decomposed the ultraviolet (UV) rays and illuminated the object with a definite wavelength. Adolf Düringer, photographer of the National Library of Vienna, in 1931 improved this process and since then works with beams of rays of the UV range. Figure 5 shows a parchment which had been written on again after the old writing has been made readable again, through the palimpsest-photography of Düringer.

Making experiments of this kind, in the year 1917 G. Kögel succeeded in preparing a paper for positive photocopies, that showed lines coloured from red-violet to brown-black on a white background. The light-sensitive substances employed for the preparation of this photocopying paper are diazo compounds. Paper is spread with them and dried in the dark. Then the copying paper is illuminated, under a transparent original, through a sunbeam or through a mercury-vapor lamp. In the non-illuminated points there is a coloured picture, that becomes visible through ammonia vapours (vapours of ammonium chloride).

The development takes place in a closed box, in which a little cup with ammonia has been put. The process lasts only a few minutes. One obtains a positive picture of the original. The tonality of the drawing depends on the choice of the coupling compound (dyestuff). One can obtain red, brown, sepia, blue and black lines on a white background. Mostly people make the so-called red copies. Paper is very sensitive and has a great resistance.

The Kögel's process has been adopted by the dyes factory Kalle and Co. of Biebrich am Rhein and introduced successfully into practical use. Already in the year 1923 it was possible to put the first photocopying paper on the market, produced on an industrial scale, for positive copies, dry, ready for development. It received the name ozalid. This word derives from the inversion of the word diazo, to which an l has been added for phonetic reasons.

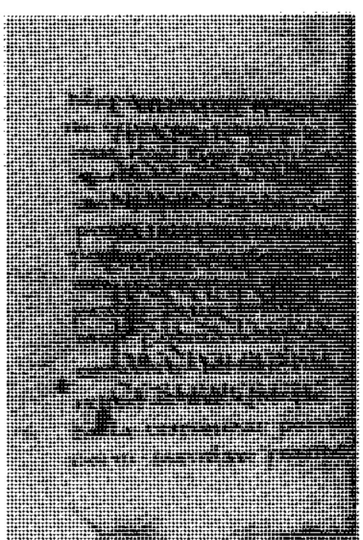

Fig. 5: Palimpsest which has been scratched out and written on again.

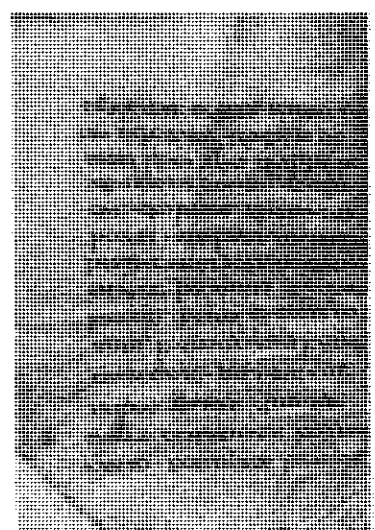

Fig. 6: Scratched out writing, made visible again by means of palimpsest-photography.

TRANSLATOR'S NOTE

This article is a translation of: "Nedoluha, Alois: Kulturgeschichte des Technischen Zeichnens", III. Teil, pages 121-126, published in: Blätter für Technikgeschichte, Heft 21 by the Technisches Museum für Industrie und Gewerbe in Vienna - Forschunginstitut für Technikgeschichte - in 1959.

The collection of papers of Ing. Nedoluha on the cultural history of engineering drawing appeared in the form of a book too: "A. Nedoluha: Kulturgeschichte des Technischen Zeichnens", published in 1960 and distributed by Springer-Verlag of Vienna.

Apart from Booker's "A History of Engineering Drawing", republished in 1979 (Northgate Co., London), almost certainly this book is the only one on the market on this subject for the time being. It should well deserve a translation into English in order to reach a greater circulation among people interested in engineering design graphics.

A noteworthy feature of Nedoluha's work is the treatment of the historical development of Technical Drawing in its interrelations with Strength of Materials, Applied Mechanics and Fabrication Processes. A whole chapter is devoted to the connections between the historical progress of metrology and the progress of the engineering graphic language.

The translator sincerely thanks Dr. Ing. Susanna Trombettoni for her collaboration in the translation from German, and expresses his gratitude to Dipl.-Ing. R. Niederhuemer, Director of the "Technisches Museum für Industrie and Gewerbe" in Vienna, for his kind permission to translate and publish some pages of Nedoluha's work.

GRETCHEN WEBER IS BACK!!

Look for her CALLIGRAPHY
on the
INSIDE BACK COVER

"THE COMMON PERPENDICULAR"

(A translation from the French of Gaspard Monge's Geometrie Descriptive, pages 37, 38 and 39, Fifth Edition, 1827)

TRANSLATED FROM THE FRENCH BY:

TERRY S. REYNOLDS, ASSOCIATE PROFESSOR
DEAN FRED O. LEIDEL, PROFESSOR
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Editor's Note: This article and the following article ("New Thoughts on Shortest Controlled Connectors", Ying and Leidel) present a new look at an old subject. Both articles should be read and savored by the reader -- especially that reader who is fascinated by the theory and application of Descriptive Geometry.

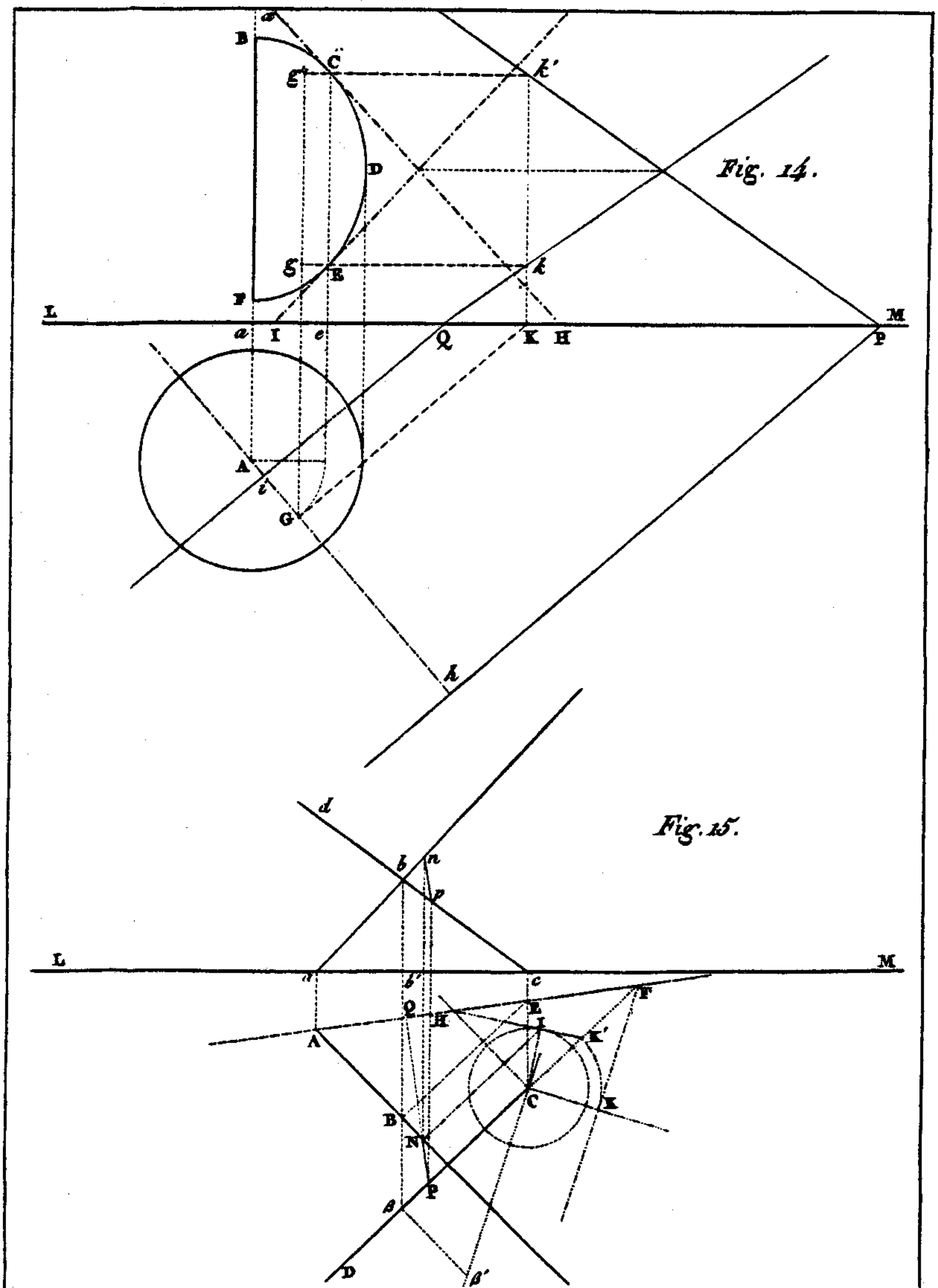
"31. Fourth question. Two straight lines being given (fig. 15), by their horizontal projections AB, CD, and by their vertical projections ab, cd, construct the projections PN, pn of the shortest possible distance, that is construct the striaght line that is at the same time perpendicular to both, and find the magnitude of this distance.

"Solution. Through the first of the two straight lines which are given let us imagine a plane parallel to the second, which is always possible, since if through any given point of the first we draw a straight line parallel to the second, and if we imagine that this third line is moved parallel to itself the length of the first, it will produce the plane in question. Let us imagine, further, a cylindrical surface with a circular base, that has for its axis the second line given, and for a radius the distance sought; this surface will be touched by the plane in a straight line that will be parallel to the axis and which will touch the first line at a point. If through this point we draw a perpendicular to the plane, it will be the line required; for it will pass, in fact, through a point of the first line given, and it will be perpendicular to it, since it will be perpendicular to a plane that passes through this line; it will touch, moreover, the second perpendicular line, since it will be a radius of the cylinder which has the second line as its axis.

"It is, therefore, only a question of constructing successively all the parts of this solution.

"1. In order to construct the traces of the plane parallel to the two lines given we will draw from any point on the first a parallel to the second; the projections of this parallel will be parallel to the lines CD,cd. The line cd touching the line ab at point b, if we drop from this point the perpendicular bb'B to the fold line LM of the planes of projection, and if we draw through the point of the first line, whose projections are B and b, the parallel to the second line, this parallel will have its horizontal and vertical projections.

Géométrie descriptive *Planche V.*



(p. 38):

" . . . the lines BE, cd; it will pierce the horizontal plane at the point E, which we obtained in drawing the line cE perpendicular to the fold line LM. Therefore, if we join the points A and E by a line, this line will be the trace of the plane parallel to the two lines given.

"2. In order to construct the line of intersection of the plane parallel to the two given lines and the cylindrical surface, it is necessary to note that this line of intersection is parallel to the second line given, and that a single point of this line determines its position. In order to find this point, we draw from any point of the second line which is the axis of the cylinder (for example, from the point C, where it strikes the horizontal plane), a plane perpendicular to this axis; the intersection of this plane with the parallel of the two lines will be tangent to the circular base of the cylindrical surface.

"The vertical plane CD having been rotated about its trace CD to the horizontal plane, we will construct the angle B'CB that the second line given makes with the horizontal plane, by taking a vertical B'B equal to b'b. The same vertical plane CD intersects the plane parallel to the two lines, along the line FK parallel to CB'. From which it follows that the plane perpendicular to the axis of the cylinder drawn from the point C strikes the vertical plane CD along the line CK perpendicular to CB' or to FK, and the horizontal plane along the line CH perpendicular to CD.

"This plane perpendicular to the axis of the cylinder, rotating around on its horizontal trace CH in order to coincide with the horizontal plane, the point K is rotated to K'; the point H of the line AE remains fixed, the line H K is the intersection of the plane tangent to the cylindrical surface and the plane perpendicular to the axis of its surface. Therefore, if from the point C we draw the perpendicular CI on its line HK', the circle described with the point C as its center, with the radius CI, is the base of the cylindrical surface, and the line IN, parallel to CD, is the horizontal projection of the line of tangency. This line touches the first line at a point whose projections are N and n, and through which passes the perpendicular to the two given lines.

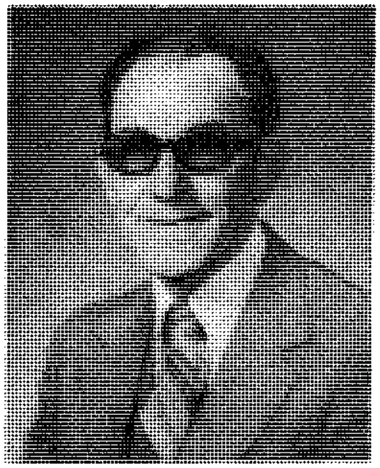
"(p. 39):

"3. Knowing the projections N, n of one of the points of the common perpendicular required, in order to have that of its perpendicular, it will suffice to draw, through point N, the line NPQ perpendicular to its trace AE. This line strikes the horizontal projection CD of the second line given at the point P, the extremity of the horizontal projection NP of the perpendicular required. The vertical projection of this perpendicular being np, we will then lay out its length by the procedure of fig. 3, plate 1.

"Consideration of a cylindrical surface touched by a plane is not necessary for the solution of the preceding problem. After having imagined a plane parallel to the two lines given, we would be able, through each of these lines, to draw to that plane a perpendicular plane, and the intersection of these two last planes would have been the direction of the required shortest distance. We are satisfied to demonstrate to you this second method, in advising the reader to seek this construction as an exercise."

1. See also "Gaspard Monge and the Origins of Descriptive Geometry," page 14 of Engineering Design Graphics Journal, Spring, 1976.

NEW THOUGHTS ON SHORTEST CONTROLLED CONNECTORS



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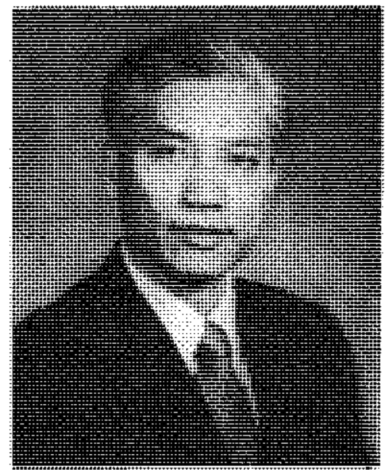

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INTRODUCTION

There are two standard analyses for the solution of the problem known as the common perpendicular problem, that of finding the shortest connector of, or the clearance between, two non-parallel non-intersecting (or skew) lines. In the most general case, both solutions originate in second auxiliary views.

The most-used solution is sometimes called the line method, and requires the point view of either of the two skew lines. It depends upon the observation that a right angle appears as a right angle if one side appears true length. It is the simplest, most straight-forward of the two standard solutions.

The alternate solution is sometimes called the plane method. It requires the true-size view of the two parallel planes determined by the two skew lines. While it is the more complex of the two standard solutions it is also a fundamental portion of the solution of other related problems such as the shortest horizontal connector of two skew lines and the shortest given-slope (or given-grade) connector of two skew lines.

As he did for all solutions that he proposed, Gaspard Monge, the originator of descriptive geometry, solved the common perpendicular problem in the top and front views.(1)* Apparently he did not recognize that other controlled connector problems existed.

Monge's solution is quite complicated, and employs rotation as well as an unidentified auxiliary view, although he may have perceived the auxiliary view as a rotation.

An orthographic drawing of Monge's solution converted to third angle projection and labeled by University of Wisconsin-Madison standards (2) is presented in Figure 1. The analysis is that one end of the common perpendicular is on the surface

* Numbers in () refer to footnotes at the end of the paper.

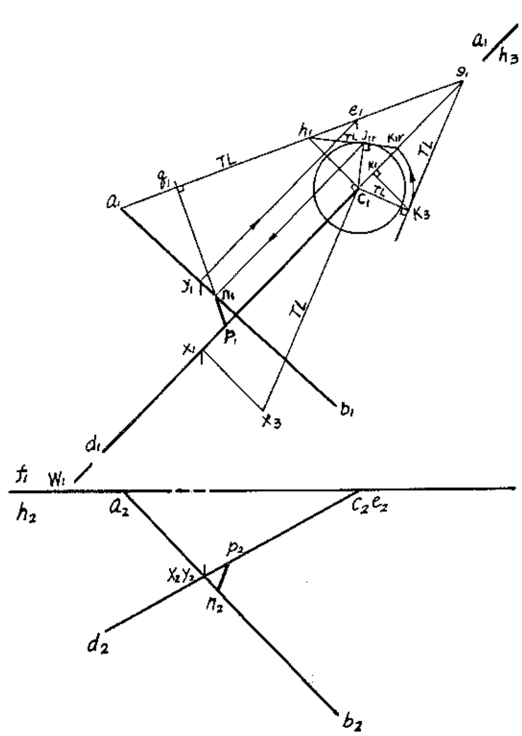


Figure 1

of the circular cylinder having one of the skew lines as its axis and tangent to the other skew line. The common perpendicular is a radial line of the cylinder.

The given skew lines are AB and CD. Plane ABE is constructed to contain line AB and be parallel to line CD, by constructing y_1e_1 parallel to c_1d_1. Arbitrarily, y_2 (and x_2) were selected as the intersection of a_2b_2 and c_2d_2. Next, consider c_1d_1 as the edge view of a plane W. Line XC, an arbitrary portion of line CD, appears true length as auxiliary view x_3c_3. For this auxiliary view, fold line a_1h_3 coincides with line c_1d_1. Line GK is the line of intersection of the plane ABE and plane W. Since plane ABE is parallel to line CD and lines CX and GK are both in plane W, therefore line GK is parallel to line CX, and can therefore be constructed parallel as line g_3k_3.

Plane CKH is the plane of right section of the analysis cylinder, and CH is its horizontal trace. Rotating plane CKH about axis CH produces $c_1h_1k_{1r}$ as the plane true size. Line KH is the line of intersection of the plane of the right section of the cylinder and plane ABE. Therefore, the circle centered at K and tangent to line HK at J is that right section, and line JN is the line of tangency of the cylinder and plane ABE. Because of the smallness of the construction, point j_1 was omitted, but it lies along line j_{1r} n_1. Point N which is on line AB, is one end of the common perpendicular.

Finally, since common perpendicular PN is perpendicular to plane ABE, p_1n_1 is drawn perpendicular to true length line a_1g_1 of plane ABE, to locate point P on line CD. (A line perpendicular to a plane is perpendicular to every line on the plane, and a right angle appears as a right angle if one side appears true length.)

Figure 2 is a pictorial view of the straight lines and planes of figure 1, but flipped upside down to place the horizontal plane H at the bottom rather than at the top, which results in maximum visibility.

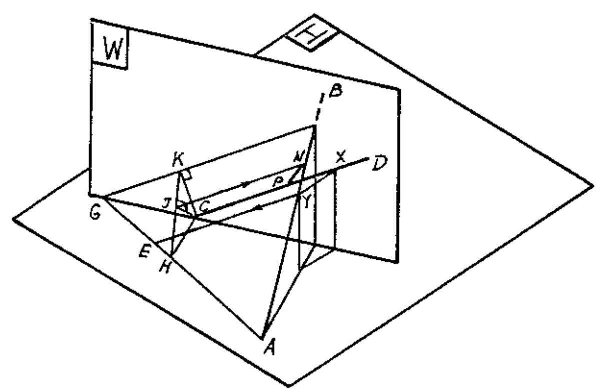


Figure 2

Within the knowledge of the authors, no "two-view" solutions of the common perpendicular or shortest controlled connector problems have been proposed in modern times. The following solutions are "two-view" solutions, and dependent upon observations of the completed solutions by the plane method. For the family of connectors that includes the shortest connector, the shortest horizontal connector, and the shortest given-slope or given grade connector, the most vital observation is that the top views of these connectors will appear perpendicular to the strike lines of the parallel planes determined by the two skew lines. Therefore in space, the connectors are indeed perpendicular to the strike lines.

COMMON PERPENDICULAR (PICTORIAL)

With the above brief background, next refer to figure 3. This is the pictorial analysis of a two-view method for finding the common perpendicular or shortest connector of two skew lines. The given skew lines are AB and CD. Plane R contains the line CD and is parallel to line AB. It was established by producing the line CE parallel to line AB. Plane Q contains the line AB and is perpendicular to plane R. It was established by producing the line BG perpendicular to plane R. Point P is the piercing point of line BG and plane R. Since the line of intersection of two planes is parallel to lines of each plane that are parallel to each other, line PZ drawn parallel to lines AB and CE is the line of intersection of planes Q and R, and crosses line CD at point X.

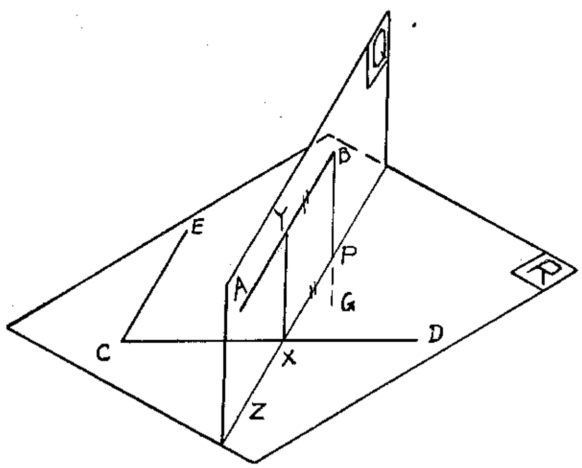


Figure 3

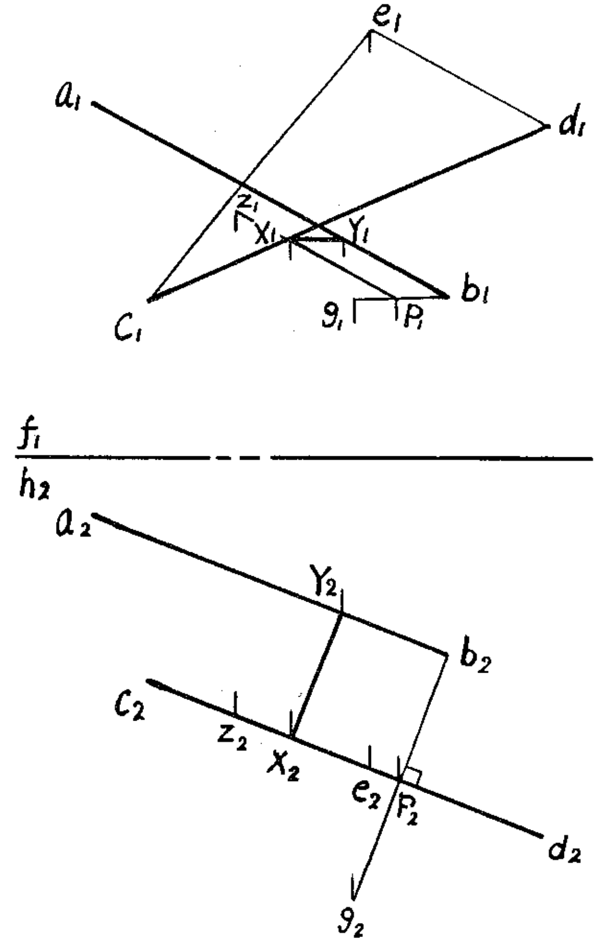


Figure 4

Since line BP is, in fact, perpendicular to both lines AB and CD, line XY drawn parallel to line BP is not only perpendicular to but also intersects lines AB and CD, and is therefore the shortest connector of lines AB and CD.

COMMON PERPENDICULAR (ORTHOGRAPHIC UNIQUE CASE)

The same problem is illustrated by the orthographic top and front views of figure 4. For maximum clarity of solution, the problem is not given as a general case; rather, skew lines AB and CD are given in the position of appearing parallel in the front view. The explanation of figure 1 applies. Planes R and Q are not shown except by the lines which lie on these planes, and line segments PG and YZ are superfluous.

COMMON PERPENDICULAR (ORTHOGRAPHIC GENERAL CASE)

The same problem is again illustrated by the orthographic top and front views of figure 5. Here, the problem is given as the general case, with AB and CD not appearing either true length or parallel in either given view. The explanation of figure 3 still applies. As in figure 4, planes R and Q are not shown except by the lines on them. Line segments PG and XZ are not shown because they are superfluous.

There are two additional features of the presentation of the problem in the general case compared with the specific case of figure 3. First, line b_1g_1 is drawn perpendicular to strike line c_1s_1 of plane CDE, in the top view, and b_2g_2 is drawn perpendicular to TL line d_2r_2 in the front view.

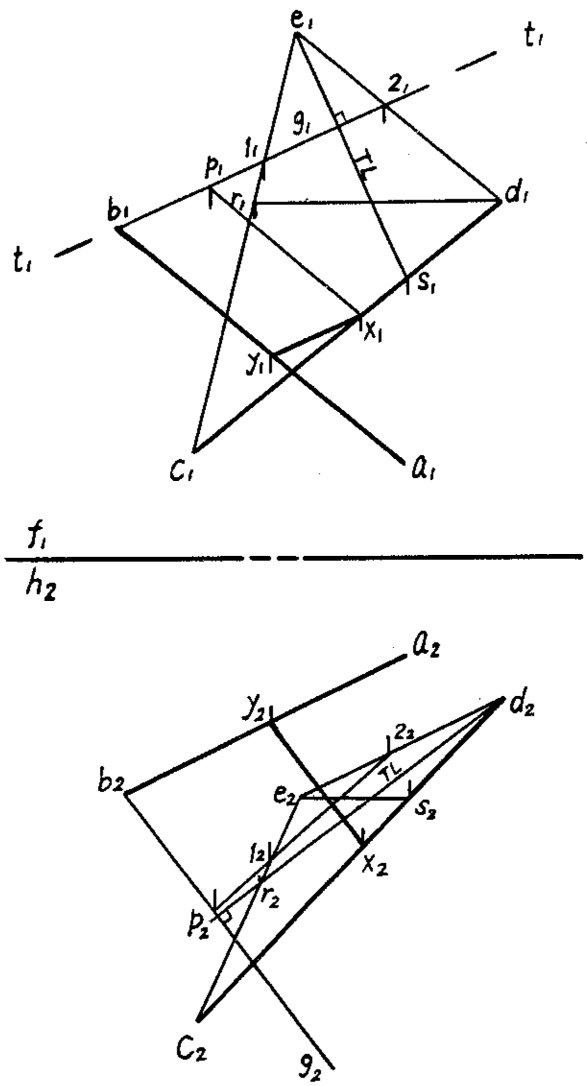


Figure 5

Second, the piercing point P of line BG and plane CDE (the plane R of figure 3) was found by the two-view method of passing cutting plane TT, which intersects plane CDE along line 1-2. Line $1_2 2_2$ intersects line $b_2 g_2$ at piercing point p_2.

SHORTEST HORIZONTAL CONNECTOR (PICTORIAL)

Figure 6 is a pictorial view illustrating the method of solution for the shortest horizontal connector of skewed lines AB and CD. The basic analysis is that the shortest horizontal connector of two skewed lines is a strike line UW of the second of two planes. The first plane CDE contains given line CD and line CE constructed parallel to given line AB, and has as a constructed strike line DG.

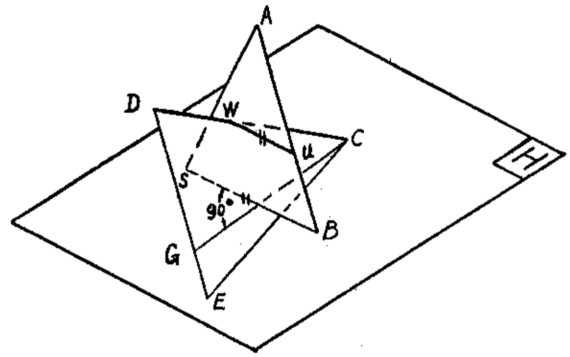


Figure 6

The second plane contains given line AB and strike line BS constructed perpendicular to but not necessarily intersecting strike line DG. Line CD pierces plane ABS at point W. It is possible to construct line WU from line CD to line AB and parallel to line BS. Line WU is the shortest horizontal connector of the two given skew lines.

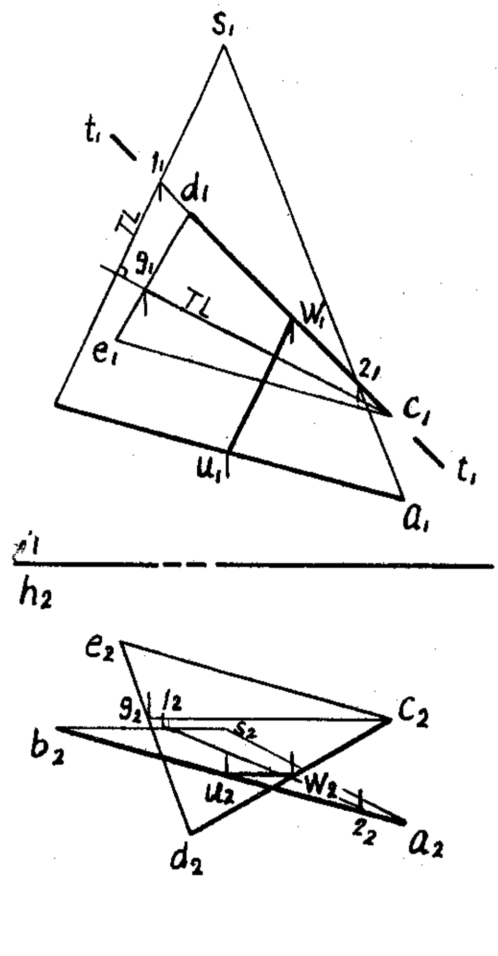


Figure 7

SHORTEST HORIZONTAL CONNECTOR - (ORTHOGRAPHIC)

The same problem is illustrated by the orthographic top and front views of figure 7. The explanation of figure 6 applies. Piercing point W was found by the two-view method, using cutting plane TT to produce line 1-2 on plane ABS. Line $1_1 2_2$ intersects line $c_2 d_2$ at piercing point W_2.

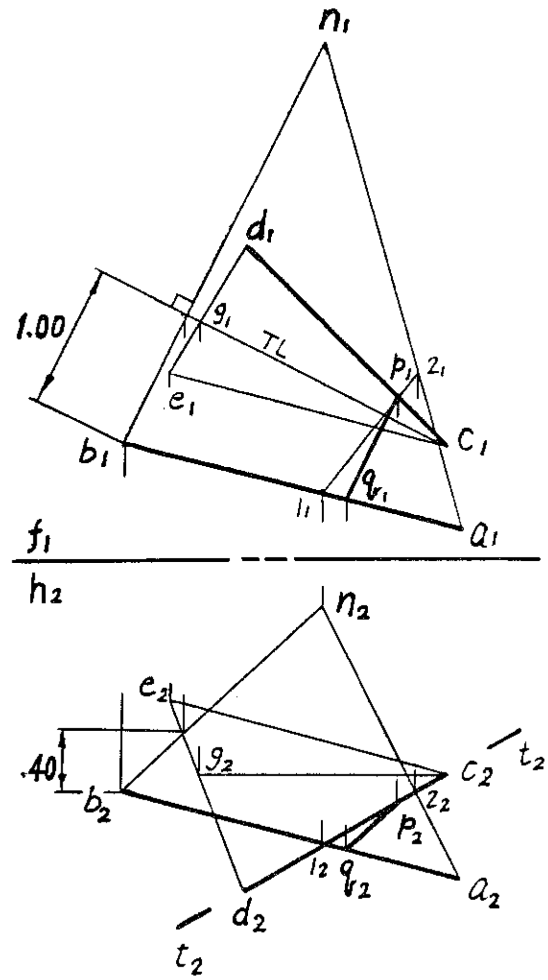


Figure 8

SHORTEST SPECIFIED GRADE CONNECTOR - (ORTHOGRAPHIC)

Figure 8 is a modification of figure 7, to show the orthographic top and front view of given skew lines AB and CD, and the solution to find the shortest 40% grade connector of the given skew lines, rather than the shortest horizontal connector. The only modification is that the second plane is constructed with given line AB, and a 40% grade line BG in place of strike line BS. In all other respects figures 7 and 8 have the same explanation, and line WU becomes the shortest 40% grade connector. If the problem required the shortest 22° slope connector rather than the shortest 40% grade connector, the 22° slope line BG could be constructed in a rotated front view.

CONCLUSION

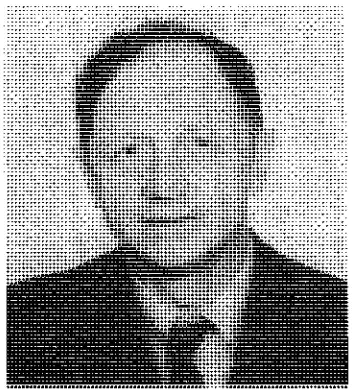
In conclusion, practical two-view solutions for the family of shortest connectors of two skew lines do exist. For anyone who truly understands the geometry of the solutions, the solutions are simple, resulting in an economy of construction time and drawing space. Just as these solutions were inspired by a re-reading of Monge, so might new solutions of other problems be similarly inspired.

FOOTNOTES

1. See "THE TRANSLATION OF MONGE'S COMMON PERPENDICULAR SOLUTION" by Reynolds & Leidel, pp - .

2. Capital letters (A,B, etc.) refer to a point in space. Lower case subscripted letters (a_1, b_2, etc.) refer to an orthographic view of a point. Top views are subscripted 1. Front views are subscripted 2. (Beyond that, view subscripts are sequential in the order of view construction.) We believe the labeling of the fold line is unique to the University of Wisconsin-Madison, superior to other systems, and most understandable to students.

GRAPHICAL SOLUTIONS IN STRUCTURAL DESIGN - MOMENT DISTRIBUTION



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Having established the moment diagrams for fully fixed ends of all the individual spans belonging to a continuous system as demonstrated in the previous article we can now release the artificial restraint of total fixity. This results in a chain reaction of re-adjustments, which was analytically demonstrated by Prof. Hardy-Cross in his famous paper on "Moment-Distribution" in 1933 and which revolutionized the design of continuous structures. Whether graphical methods existed at that time or whether their introduction was spawned by that event seems not clear. This author received instructions on the subject during his engineering education in the late 1930's and used them in practice. Some of such solutions are also described by D.B. Steinman in his article on the subject in "Engineering News Record" of June 5, 1944. Otherwise, the procedures seem to be unknown or will soon be forgotten.

Those who are familiar with the theories of structures know that the areas and the end reactions of the moment diagrams in Fig. 1a and 1b in the previous article are equal. Also, that a change in the degree of restraint on one side, will change the moments at the supports but not the reaction at the unchanged end for the negative moment area assumed as an area load. Graphically, this may be demonstrated as follows:

Removing the restraint at one support only or changing the degree of restraint at one support will cause a change in the mid-third point ordinate adjacent to that support while the other ordinate remains unchanged. In other words the closing line rotates about 'ℓ' if the degree of restraing changes at 'B' and about 'r' if the change is at 'A'. (See Figure 17.) If M_A changes to 0, M_B will increase to $M_B + \frac{1}{2}M_A$ and if we remove the restraing at B to $M_B = 0$, M_A will become $M_A + \frac{1}{2}M_B$. If M_A is reduced to $\frac{1}{2}M_A$, M_B will increase to $M_B + \frac{1}{4}M_A$. Thus we receive graphically the carry-over factor. (See Figure 18.)

If M_B reduces, M_A increases by $\frac{1}{2}$ of the reduction because the closing line rotates about 'ℓ'.

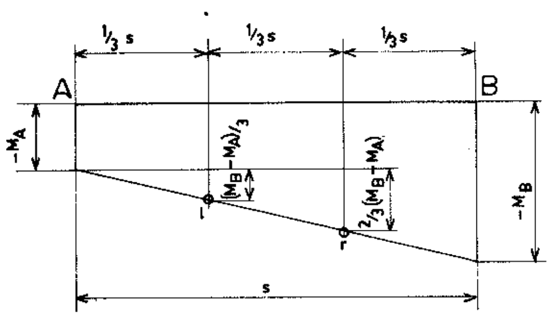

FIG. 17

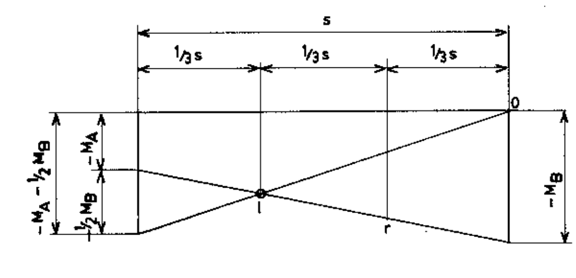

FIG. 18

Continuous Beams

A beam with fixed ends and one intermediate support is in balance from the start if $s_{A-B} = s_{B-C}$, $I_{A-B} = I_{B-C}$ and the loading is symmetrically arranged about 'B' (see Figure 19a.) $M_A = M_{Bℓ} = M_{Br} = M_C$. If we remove the restraints at A and C, both ends now being simply supported, the closing lines will rotate about 'r' in the left span and about 'ℓ' in the right span until both M_A and M_C, are "0" (see Figure 19a.) $M_{Bℓ}$ and M_{Br} are still equal in magnitude but they have both increased by $\frac{1}{2}$ of the original M_A and of the original M_C respectively. Because of this equality of the moments on either side of 'B' the system is in balance.

If we remove the restraining at one support (say at 'A') only, see Figure 19c, the closing line in the left span would rotate about "r". Since $M_{Bℓ}$ and M_{Br} must balance, the difference "y" must be distributed in proportion to the respective stiffnesses of the two spans A-B and B-C. The closing line 2-3 will rotate about "r" of the right span. Since "A" is not restrained, M_A = "0" and $\frac{1}{2}$ of the carry-over or $\frac{1}{4}$ of the movement of $M_{Bℓ}$ will reflect back to "Bℓ". This operation is simplified by stating that if the rigidity of a beam restrained at the end is I/s, the rigidity of a

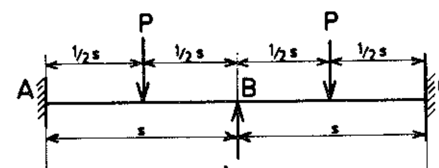

FIG.19a

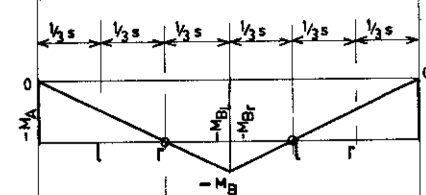

FIG.19b

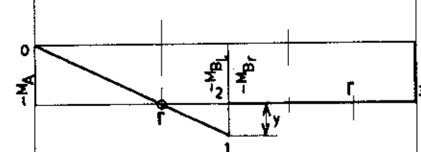

FIG.19c

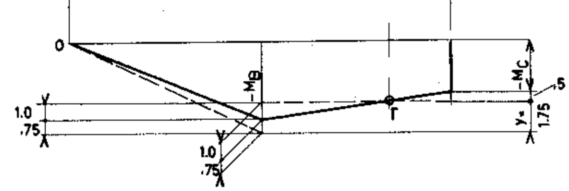

FIG.19d

beam with a freely supported or hinged end is only $3I/4s$ (span A-B in Figure 19d). In the example in Figure 19d, $I_{A-B} = 3/4$, $I_{B-C} = 1$ and $s_{A-B} = s_{B-C}$. If "y" = 1.75, point "2" moves 1.0 down and the closing line rotates about "r". Point 3 at M_C moves up 0.5 reducing M_C by 0.5. Point "1" moves up 0.75 to meet point 2, the line 0-1 rotating about "0" (see Figure 19d). 'I' = moment of inertia for the trial section.

The general case of graphical moment distribution for a two span continuous beam is shown in Figure 20. The supports at "A" and "C" are fixed.

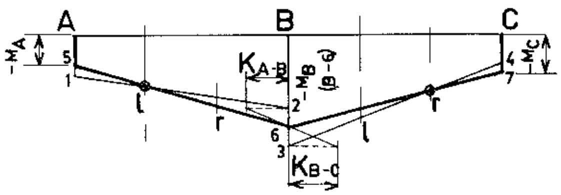

FIG.20

Initial fixed-end moments and closing lines are first established. In this example they are 1-2 and 3-4. The relative rigidities K_{A-B} and K_{B-C} ($K_{A-B} = I_{A-B}$ to s_{A-B} ; $K_{B-C} = I_{B-C}$ to s_{B-C}) are drawn to any convenient scale as indicated by the two dashed lines. (See Figure 20.) A line connecting the end points of these two lines divides the moment imbalance 2-3 in proportion to the relative rigidities of the two beams. The final closing lines from '6' through 'ℓ' in the left span and from '6' through 'r' in the right span establish the final negative moments. Note that the carry-over 1-5 = ½ of 2-6 and 4-7 = ½ of 3-6.

The accuracy of the drawing can be improved by exaggerating the vertical moment scale. The scale of the span length is <u>not</u> important; indeed, the span length may <u>not be</u> to scale. The important thing is that the 'ℓ' and 'r' - points are located exactly in the 1/3 - points of each span. If that is so, we may draw the span lengths in proportion to their relative rigidities and save some steps in the procedure. In Figure 21, the span lengths are measured as relative

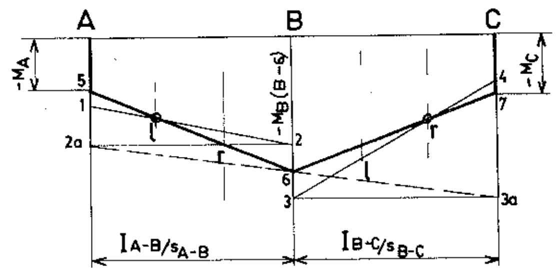

FIG. 21

values of I/s. The closing lines for fixed end moments are 1-2 and 3-4. Projections 2-2a and 3-3a are parallel with the base line A-B-C. The dashed line 2a-3a intersects the imbalance of the initial moments 2-3 under "B" at "6". Lines 6-5 through "ℓ" in the left span and 6-7 through "r" in the right span produce the final closing lines and end-moments.

In Figure 22, a beam with two intermediate supports is shown. "A" is simply supported, "D" is fixed. Span lengths need not be to scale. Moments must be to scale as well as the stiffness factors "K".

<u>Procedure:</u>

(1) Draw initial fixed end moments to scale. Their closing lines are shown dashed. Since "A" is hinged, M_A has to be zero. The closing line for the initial moment has to go from "0" at "A" through "r_{A-B}" to "1" at "B".

(2) Construct points "m" and "n" by drawing K_{A-B} over "B", (3/4 K_{A-B}

if "A" is hinged), $\frac{1}{2}$ K_{B-C} under the 1/3 points of span B-C and "K_{C-D}" over "C". The points "m" and "n" are the intersections of the stiffness factor-end point connections with the base line as shown in Figure 22.

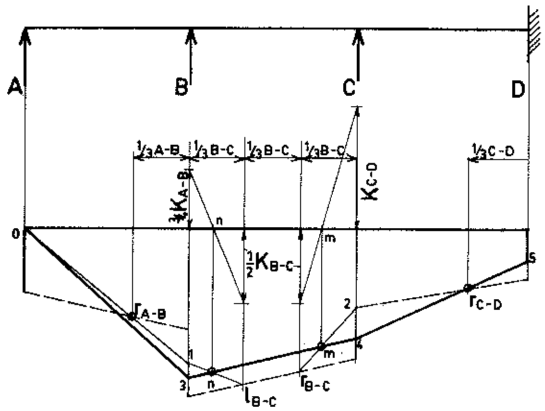

FIG. 22

(3) Since "ℓ_{B-C}" and "r_{B-C}" are rotating points of the closing line, line 1-ℓ_{B-C} and 2-r_{B-C} are possible closing lines considering only spans A-B-C or B-C-D respectively. Their intersections with points "m" and "n" projected down from the base line are 2 points on the final closing line 3-4 in span B-C. M_A = 0; 0-3 is therefore the final closing line for the moment diagram in span A-B. "D" is fixed. The final closing line in span C-D has to rotate about "r_{C-D}" to 4-5.

The final negative moments are M_A = 0; $-M_B$ = B-3; $-M_C$ = C-4; $-M_D$ = D-5.

Figure 23 shows the solution for a continuous beam with three intermediate supports. As previously stated, spans do not need to be drawn to scale; however, moments and stiffness factors (K) must be drawn to scale. To solve the problem, the "n" and "m" rotation points are constructed in spans B-C and C-D. In these two spans intermediate closing lines "2a" directed through n-r_{B-C} and "3a" directed through m-ℓ_{C-D} establish a final moment difference over the center support "C". This difference is to be divided in proportion to the relative stiffness of K_{A-B-C} and K_{E-D-C}. The latter are established as shown along the base line between B-C and C-D. A line from "n_{B-C}" through the altitude of $\frac{1}{2}K_{B-C}$ gives K_{A-B-C} over "C" and a line from "m_{C-D}" through the altitude of $\frac{1}{2}K_{C-D}$ gives K_{E-D-C} under "C". Figure 23 shows how the moment difference may be divided (see also Figure 20). The dividing point establishes the

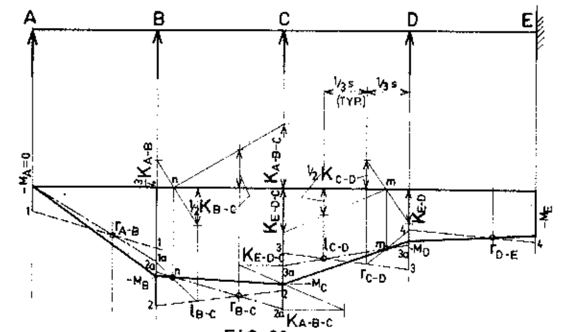

FIG. 23

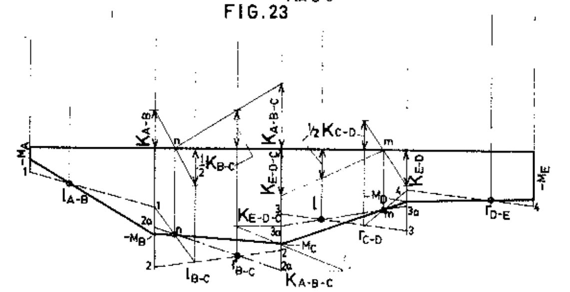

FIG. 23a

final negative moment at "C" ($-M_C$). Final closing lines may now be drawn from ($-M_C$) through "n" establishing ($-M_B$) and through "m" establishing ($-M_D$). Since in this example "A" is hinged, M_A = 0, the final closing line is directed from ($-M_B$) to "0" at "A". The final closing line D-E is directed from ($-M_D$) through "r_{D-E}" establishing ($-M_E$). If "A" were restrained, the flow of the final closing lines would be different. In Figure 23a the intermediate closing line 0-1a would not be required. "n_{B-C}" is projected to intersect with line 1-ℓ_{B-C} to establish the rotation point "n" of the closing line in span B-C. K_{A-B} has increased from 3/4 to 1 K_{A-B} because "A" is fixed. The final closing line in span A-B will point from ($-M_B$) through $_{A-B}$ to establish ($-M_A$), "A" being restrained, ($-M_B$) is somewhat less. The effect on ($-M_C$), ($-M_D$) etc. is less the further away one gets from "A".

A beam with 4 intermediate supports is shown in Figure 24.

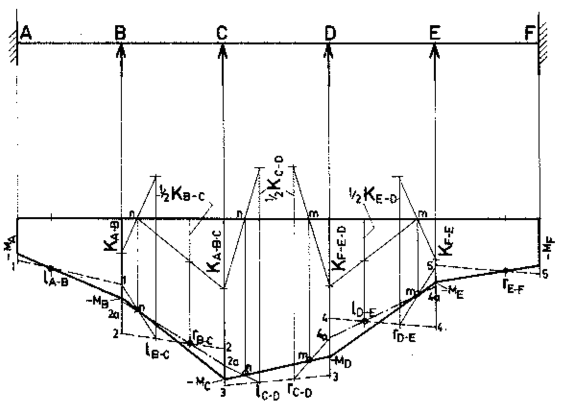

FIG. 24

The end supports are restrained. If one or both were hinged or freely supported the procedure described in connection with Figure 23 would apply. However, when compared with Figure 23, it will be noted that K_{A-B-C} appears under "C" and K_{F-E-D} under "D". The intermediate closing lines "2a" through "n_{B-C}" and "r_{B-C}" "4a" through "m_{D-E}" and "ℓ_{D-E}" establish intermediate moment differences at "C" and "D". "n_{C-D}" on line 2a-ℓ_{C-D} and "m_{C-D}" on line 4a-r_{C-D} are points of the final closing line between "C" and "D" establishing the magnitudes of the final ($-M_C$) and ($-M_D$). ($-M_C$) through "n_{B-C}" gives ($-M_B$), and ($-M_D$) through m_{D-E} establishes ($-M_E$). ($-M_B$) through ℓ_{A-B} gives ($-M_A$), and ($-M_E$) through r_{E-F} establishes ($-M_F$).

The procedure may now be extended to beams with any number of intermediate supports.

Frames

The idea of graphically distributing moments in 2 intersecting structural members whose 4 ends are fixed is shown in Figure 25. Initial negative moments are assumed in this case over the span A-D only. By releasing Point A from total restraint, the moment difference at "A" will distribute itself into all 4 members relative to their respective stiffness factors.

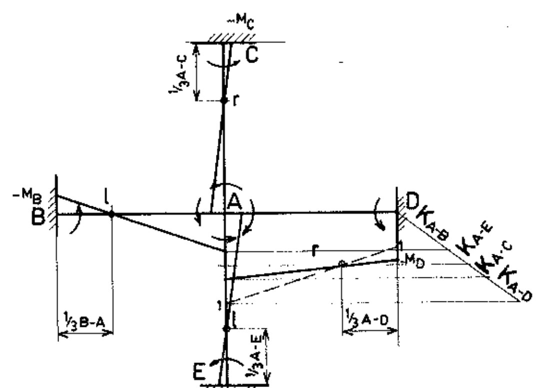

FIG.25

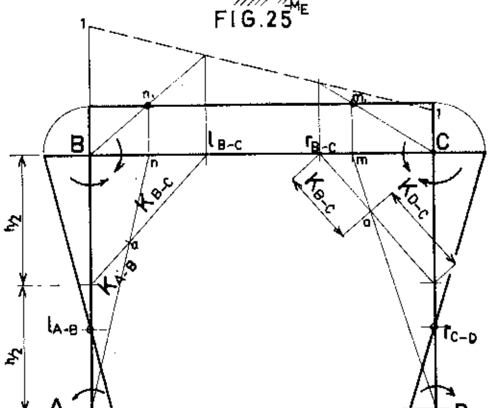

FIG.26

The initial (clockwise) action in one member is balanced by (counter-clockwise) counteraction of all members. In

Figure 25 the initial M-difference is divided relative to the stiffness factors of the 4 members to establish the magnitude of the counteraction in each member. The closing lines point through the "ℓ" and "r" points of the members at the one-third points adjacent to the fixed supports where (counter clockwise) action is produced in the form of the carry-over moments amounting to 50% of the values at "A".

Figure 26 shows a simple frame consisting of a beam supported by columns at the ends. All connections are restrained. On the drawing, only moments and stiffness factors must be to scale. The "ℓ" and "r" lines must be drawn through the third-points of the span length or the columns' heights. The system may be treated as a continuous beam and solved as shown in Figure 22; however, time may be saved by finding "n" and "m" as shown in Figure 26. The relative values of "k" are drawn from ℓ_{B-C} and r_{B-C} to any point along the column height, in Figure 26 to the $\frac{1}{2}$ point in in Figure 27 to the 1/3 point adjacent to the top. A second line is drawn from twice the distance down the column height, through the dividing point "a" of the line representing the relative stiffness factors to establish "n" and "m" on the B-C base line.

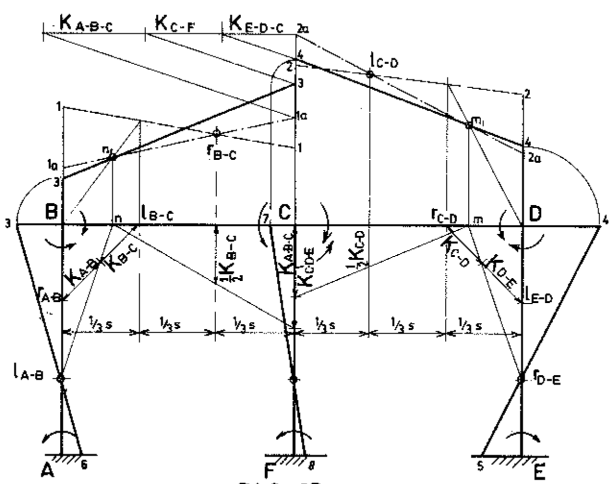

FIG. 27

Since there are no initial moments existing in the columns, a possible final closing line (considering only "A" and "B" or only "D" and "C") would extend from "B" through the intersection of the ℓ_{B-C} third-point line with the initial closing line 1-1, and another from "C" through the intersection of the r_{B-C} third-point line with line 1-1. The vertical projections of "n" and "m" onto these lines establish direction and location of the final closing line between "B" and "C". M_B and M_C of course will continue around the corners of the frame and are of the same magnitude at the top of the columns as at the ends of the span. $M_A = \frac{1}{2}M_B$ and $M_D = \frac{1}{2} M_C$ as shown in Figure 26.

In Figure 27, the previous example has been expanded with the introduction of an intermediate column. The "n" and "m" points are found as before. Intermediate closing lines "1a" and "2a" are established through $n_1 - r_{B-C}$ and $m_1 - \ell_{C-D}$. The moment difference 1a - 2a over "C" is distributed in proportion to the stiffness factors of the three members composing point "C", K_{A-B-C}, K_{C-F} and K_{E-D-C} as shown on top of Figure 27. K_{A-B-C} and K_{E-D-C} have been found as shown in previous examples below base line B-C-D.

A continuous frame with two intermediate columns is shown in Figure 28. One new step is introduced. After finding the intermediate closing lines 1a-1a in span B-C and 3a-3a in span D-E as demonstrated in Figure 27, the stiffness factors for the intermediate columns K_{C-H} and K_{D-G} are added to K_{A-B-C} and K_{F-E-D} respectively. The "n" and "m" - points in the center span are then found as shown. The intersections of the upward projections of "n" and "m" with lines

"1a" (over "C") - ℓ_{C-D} and "3a" (over "D") - "r_{C-D}" are points on the final closing line 4-4 over C-D. The final closing line over B-C would point from 4 over C through $n_{1\ B-C}$ if K_{C-H} were zero. Since this is not the case, the moment difference 4-1a over "C" must be divided in relation to the relative stiffnesses of A-B-C and C-H for example as shown in Figure 28. The final closing lines in the end spans will then follow from 5 over "C" through n_1 to "7" (the final moment at "B") and from 5 over "D" through "m_1" to "6" (the final moment at "B"). 4-5 are the final moments at the top of the intermediate columns.

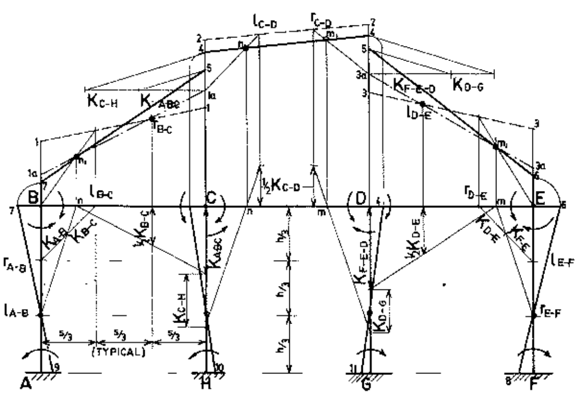

FIG. 28

M_B and M_E are also the final moments at the top of the exterior columns. The final closing lines over the column heights point through the lower mid-third points and show the 50% carry-over moments at the column bases. If any columns were hinged at the bases, their closing lines would have pointed directly to these hinges and the stiffness factors of such columns would have been only 75% of I/h.

Further development of graphical solutions for more complex structures and perhaps simpler solutions and time-saving steps are possible and hopefully encouraged by the procedures demonstrated.

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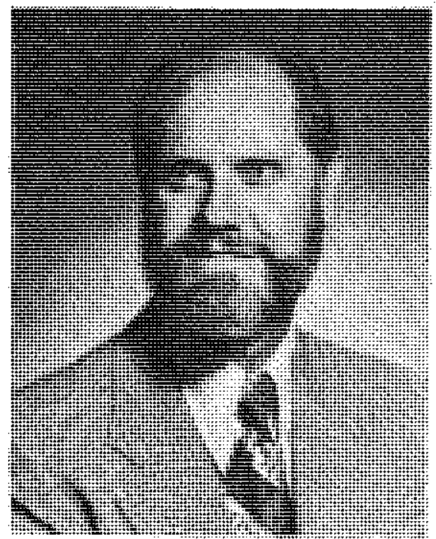
INCREASING UNDERSTANDING AND VISUALIZATION ABILITIES USING THREE-DIMENSIONAL MODELS



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INTRODUCTION

With increased professional demands placed upon teaching faculty today - research, publishing, advising and counseling, committee work, extension service, etc., - it is only natural that even the "best"teachers rely more and more on "chalk and talk" methods of instruction. Because of the many varied activities teaching personnel are expected to participate in, it is quite understandable that visual teaching aids in many instances are never seriously considered for classroom use. Truth of the matter is, however, visual aids, especially 3-dimensional models, when appropriate to the teaching situation, (1) save valuable instruction time, (2) result in increased student interest and information-retention and (3) reinforce student learning.

In retrospect, it is interesting to recall how most of us began learning as children, even as far back as crawling on our hands and knees; touching, feeling, and seeing things, mostly in three-dimensions. During the primary grades, at least through the fifth or sixth grade, pictures and 3-dimensional models played an especially important part in meaningful learning. It almost appears at times, that the higher-up the educational ladder one progresses, the fewer opportunities there are for this basic hands-on, "look-at-the-model" type of learning situation. This bit of reality is truly very unfortunate simply because of the large amount of knowledge (estimated to be 83% of the total) we assimilate through sight, and of course, by way of touch. It is due to this important valuable medium for learning-reinforcement that the author calls attention to the need for increased utilization of 3-dimensional models in the classroom and suggests a rather inexpensive and quick way of fabricating engineering drawing models from insulation foam board.

MODEL CONSTRUCTION

The following photos shown in Figures 1 through 4 illustrate the complexity of shapes that may be constructed using two-inch thick insulation foam board, rubber cement and a felt marker. Actually, any conceivable object shape or geometric configuration may be constructed by reducing the object to its basic geometric elements. In producing an object, the process may be considered and operation of either (1) adding on some required component element, (2) cutting away portions of material to represent such features as holes or slots, or (3) a combination of both of these.

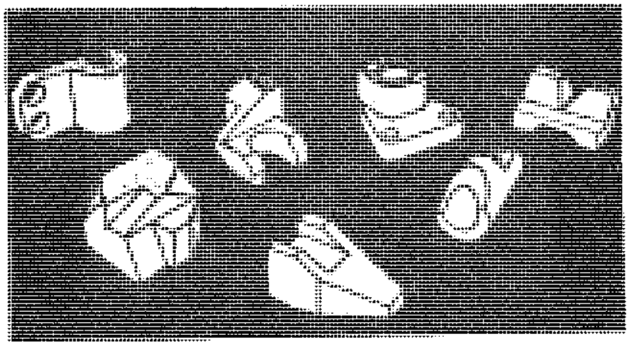

Figure 1

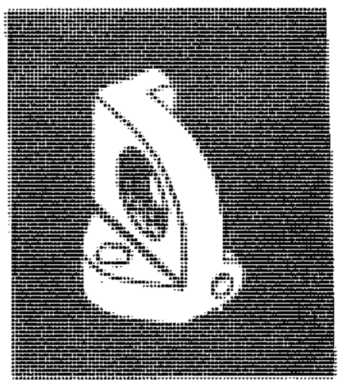

Figure 2

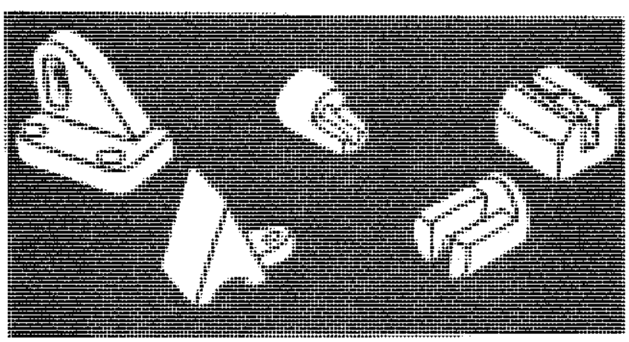

Figure 3

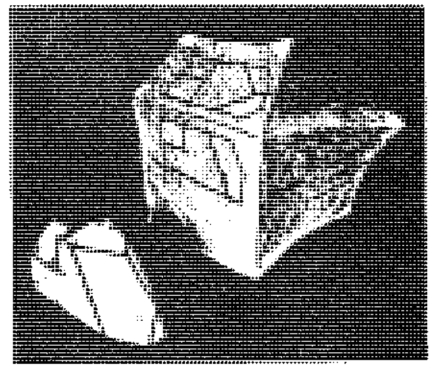

Figure 4

The equipment necessary to cut insulation foam board into variuous shapes (including minor details such as chamfers, counter-bored holes, and the like) is a band saw, or scroll saw, with an eight to ten teeth per inch saw and a good, sharp pocket or Exacto knife.

Figures 5a and 6a present photographs of two different models constructed and used in the Engineering Graphics classrooms at North Carolina State University. Shown in the photos of Figures 5b and 6b are various object elements for each of the completed objects just mentioned, prior to assembling with rubber cement. The major elements of each object have been cut on the band saw. The counter-bored and drilled-hole features have been cut from the foam board with and Exacto knife. During the assembly stage it is best to apply rubber cement to all pieces at their appropriate places of contact and then let stand until they appear dry before placing them together. This procedure will insure that when the pieces are brought together they will adhere permanently. The best time to outline the various features and edges for highlighting purposes is after the object

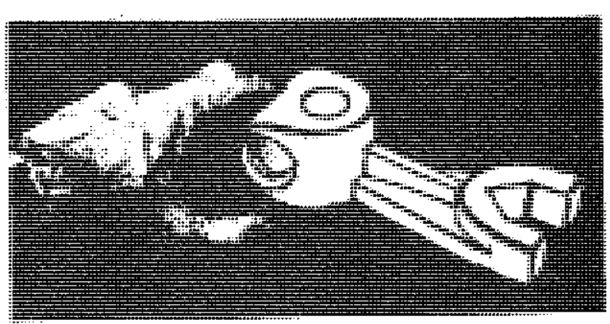

Figure 5a

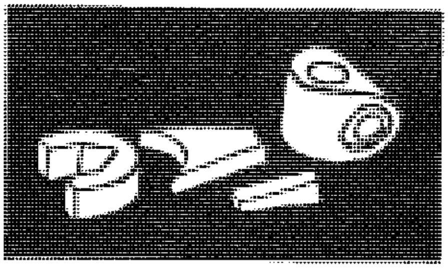

Figure 5b

has been completely assembled. The primary reason for outlining the objects, of course, is so the students may see clearly where lines should appear on their drawings, or, as the case may be, where lines should not appear.

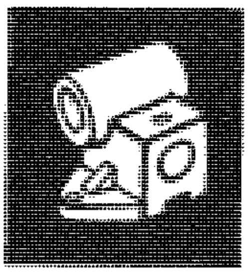

Figure 6a

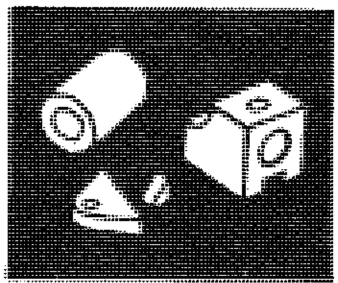

Figure 6b

Photo Figures 6a and 6b illustrate an object that required a relatively large cylinder be cut from the foam board. For this particular feature it was necessary to cement two pieces together prior to cutting the geometric shape on the band saw. In any case, regardless of the size or shape of an object element, using insulation foam board and rubber cement enables one to readily improvise prior to, or during the machining state of model construction.

REACTION TO USING 3-DIMENSIONAL MODELS

Unequivocally, the use of models similar to those previously shown and described have been very favorably received by students and teachers alike. Numerous models have been used for instructional purposes and learning reinforcement of certain principles in Isometric, Oblique, Orthographic, Sectional, and Auxiliary drawing with the same high level of success. Based upon student feedback (primarily of the direct unsolicited verbal type) the opportunity of seeing in three-dimensions, a model that they were required to describe on two-dimensional paper, not only allowed them to see the basic shape and details of the various geometric elements, but also resulted in a more thorough understanding of how and where certain lines should appear on their finished drawings. There isn't a day when these types of models are used in the classroom that one or more students do not make positive comments concerning their helpfulness to them. And on occasion, it is not uncommon to hear from the student(s) simply, "Ah-Ha!" or, "Oh, now I understand where that line comes from."

From the teacher's point of view, and without reservations, the time saved during class in describing the "whys and wherefores" of an object to a student - not to mention the true understanding that is gained - far outweighs the time and effort spent in designing and constructing such models.

SUMMARY

The author has attempted through this article to propose a closer look at the inherent value of using 3-dimensional models for learning-reinforements. Based upon personal experience in designing, fabricating, and using various geometric objects in the classroom, it has been stressed that the benefits derived from such models far outweigh the necessary work required in developing and producing them.

Methods for producing similar models rather quickly and inexpensively from insulation foam board were also explained and described. In addition, the value of using 3-dimensional models in subject areas other than Engineering Graphics was suggested. Finally, based upon positive reaction from student and teacher alike, it was concluded that more opportunity for hands-on use of three-dimensional models in the classroom would result in better student understanding of subject matter and increased ability to visualize various geometric shapes more easily.

PUZZLE CORNER

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SPRING '81 PUZZLE:

Given: Four non-parallel unlimited skew lines in general positions.

Determine: The center of the sphere which is tangent to all of the given lines.

Pleasemail solutions before October 1, 1981 to:

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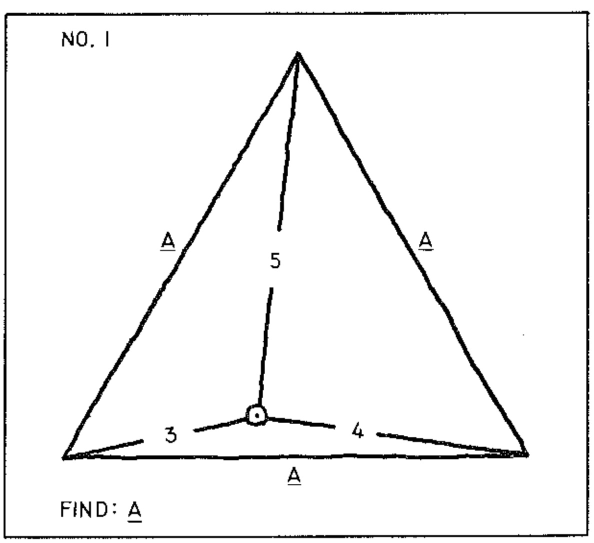
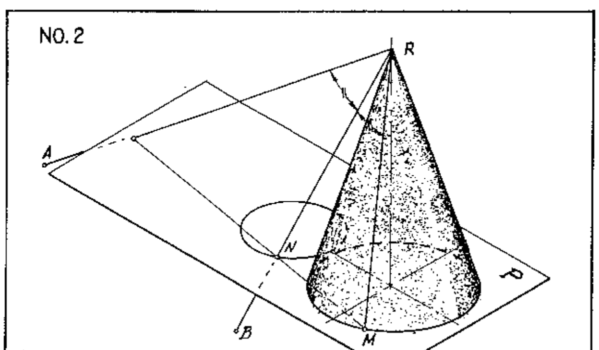
Ruston, Louisiana 71272

Below are the two puzzles repeated from the Fall '80 issue:





Shown:

- a right circular cone with its apex R & its base circle in the plane P;
- an arbitrary point M on the base circle of the cone (-the position of the point M varies);
- a fixed point A outside the cone;
- the bisector RB of the angle ARM;
- the point N of intersection of RB with the base plane P.

Required: To demonstrate (prove) that, for all points M on the base circle of the cone, the locus of points N is a circle.

The following is the calculated solution to No. 1 from Chi Di Lin.

The following is the calculated solution to No. 1 from Chi Di Lin of the Anhwei Institute of Technology, Hofei Anhwei, the P.R. of China. (Remember Chi Di Lin's beautiful solution to the Perplexahedron in the Fall '80 issue!) We wish to thank Dick Leuba of North Carolina State University for posing the problem. Chi Di Lin's answer works out to be: 6.7664 . . . Figure 1 is his geometric derivation of that length and also shows the location of the angle α and the angle β which is used in his calculation.

$$\alpha + \beta = 60^\circ$$

$$\left.\begin{aligned} 5^2 &= \underline{A}^2 + 3^2 - 6\underline{A}\cos\alpha \\ 4^2 &= \underline{A}^2 + 3^2 - 6A\cos\beta \end{aligned}\right\}\text{Law of Cosine}$$

combining these formulae and simplifying ...

$$4\underline{A}^2\cos^2\beta - 6\underline{A}\cos\beta + 3(3 - \underline{A}^2) = 0$$

applying the quadratic formula and solving for $\cos\beta$, ...

$$\cos\beta = \frac{3 \pm \sqrt{12\underline{A}^2 - 27}}{4\underline{A}}$$

Since $\underline{A} > 5$. and $\cos\beta > 0$, then

$$\cos\beta = \frac{3 + \sqrt{12\underline{A}^2 - 27}}{4\underline{A}}$$

then:

$$4^2 = A^2 + 3^2 - \frac{3(3 + \sqrt{12\underline{A}^2 - 27})}{2}$$

simplifying:

$$\underline{A}^4 - 50\underline{A}^2 + 193 = 0$$

applying the quadratic formula and solving for $\underline{A}$, ...

$$\underline{A}_{1,2} = \pm\sqrt{25 + 12\sqrt{3}}$$

$$\underline{A}_{3,4} = \pm\sqrt{25 - 12\sqrt{3}}$$

Since $\underline{A} > 5$,

$$\underline{A} = \sqrt{25 + 12\sqrt{3}} \quad \text{or}$$

$$\underline{A} = \sqrt{5^2 + (2\sqrt{3\sqrt{3}})^2}$$

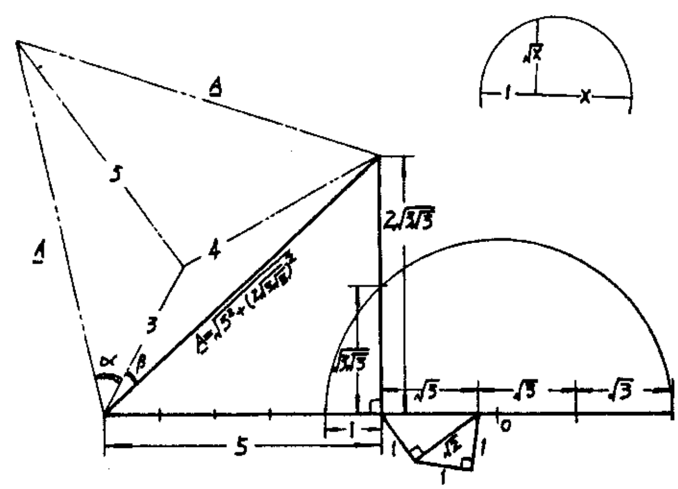
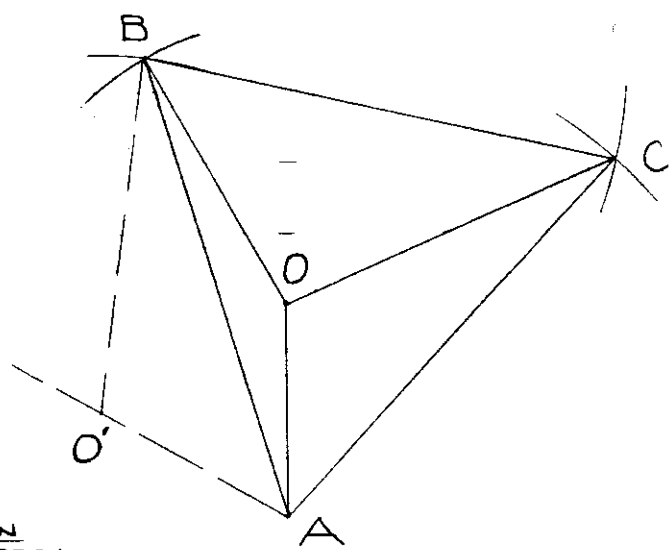


FIGURE ONE

Figure 2 is a graphical solution
by Dick Leuba to Puzzle No. 1.

GIVEN: OA = 3   OB = 4   OC = 5
ABC IS AN EQUILATERAL △

FIND: LENGTH OF AB (= BC = AC)



BEGINNING WITH A CLEAN PIECE OF PAPER:

- LAY OUT OA = 3 IN ANY DIRECTION
- LOCATE O' SUCH THAT ∠OAO' = 60° AND SO THAT AO' = 3
- CROSS TWO ARCS TO LOCATE B WITH OB = 4 & O'B = 5
- AB IS NOW ONE SIDE OF THE DESIRED EQUILATERAL △
- CROSS TWO ARCS TO LOCATE C SUCH THAT BC = AB & AC = AB
- AB (= BC = AC) = 6.7+ BY SCALE MEASURE ← ANS.

PROOF:

- OA = 3   OB = 4   O'B = 5 }
- ∠OAO' = 60°   AO' = AO } · · · BY CONSTRUCTION
- ABC IS EQUILATERAL △ }
- ∠CAB = 60°   AB = AC · · · · PROPERTY OF EQUILATERAL △
- ∠OAC = ∠O'AB · · · · · · · · · {∠OAC = 60° − ∠OAB; ∠O'AB = 60° − ∠OAB}
- △AO'B ≅ △AOC · · · · · · · · · · SIDE-ANGLE-SIDE
- OC = 5 · · · · · · · · · · · · · · · · CONGRUENT △S

FIGURE TWO

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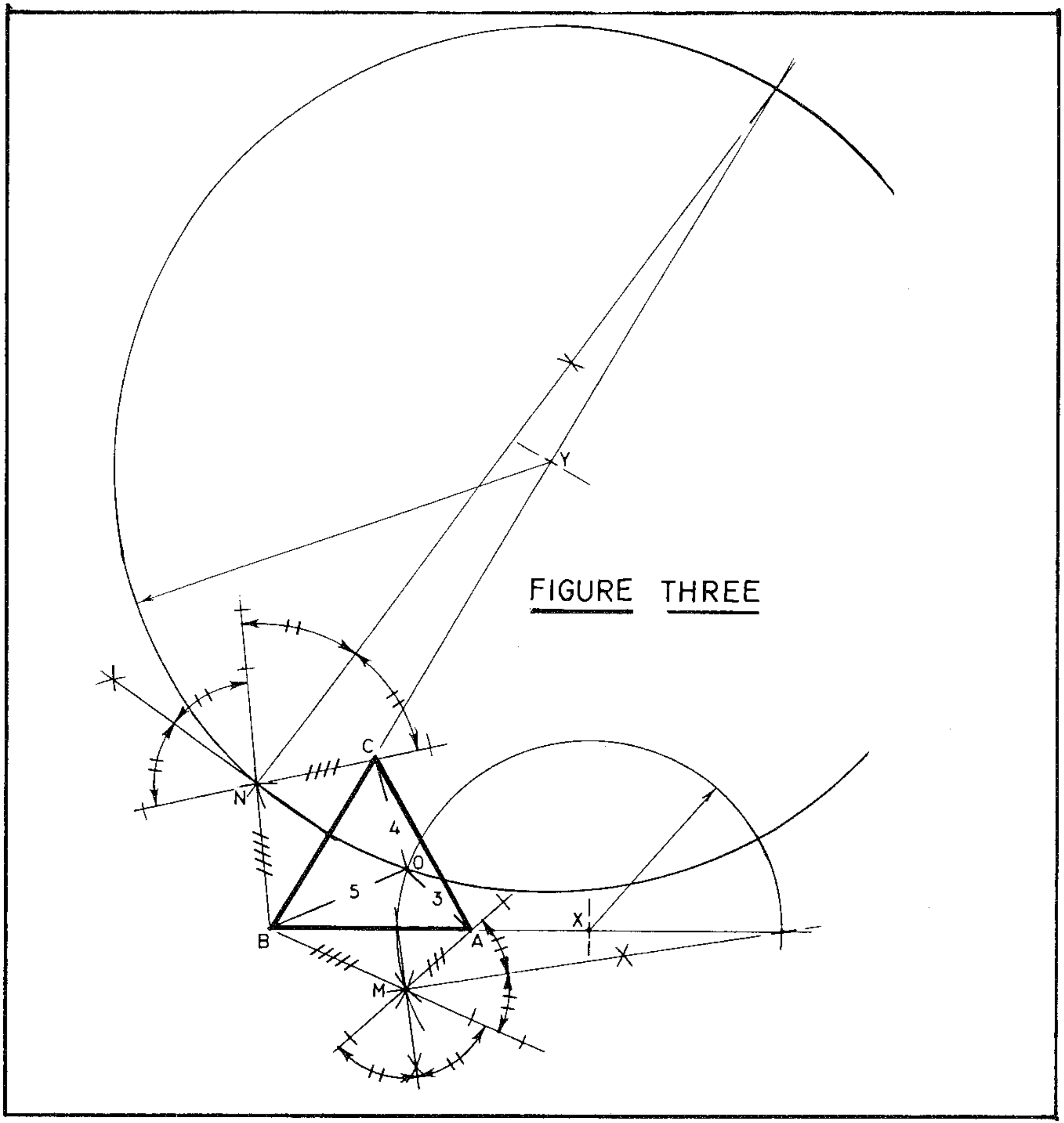


Figure 3 is the 'Corner's graphical solution to puzzle No. 1. It employs the principle: The locus of a point, say M or N, is a circle if its respective distances from two fixed points is in a constant ratio (excepting: 1:1). If A and B are the fixed points, a circle with center at X and radius XM may be constructed which contains all the points which have the constant distances ratio of 3:5 from the points A and B respectively. (See the Circle of Apallonius in your favorite geometry text). Similarly, a circle may be constructed with center at Y and the radius YN which contains all the points which have the constant distances ratio of 5:4 from the points B and C respectively. The intersection O of these circles is the requisite point. Obviously, this method has nothing to recommend it over Leuba's.

Figure 4 and explanation are from Chi Di Lin and are the solution to puzzle No. 2. The puzzle was posed by Abe Rotenberg or the University of Melbourne, Australia, and we thank him for it.

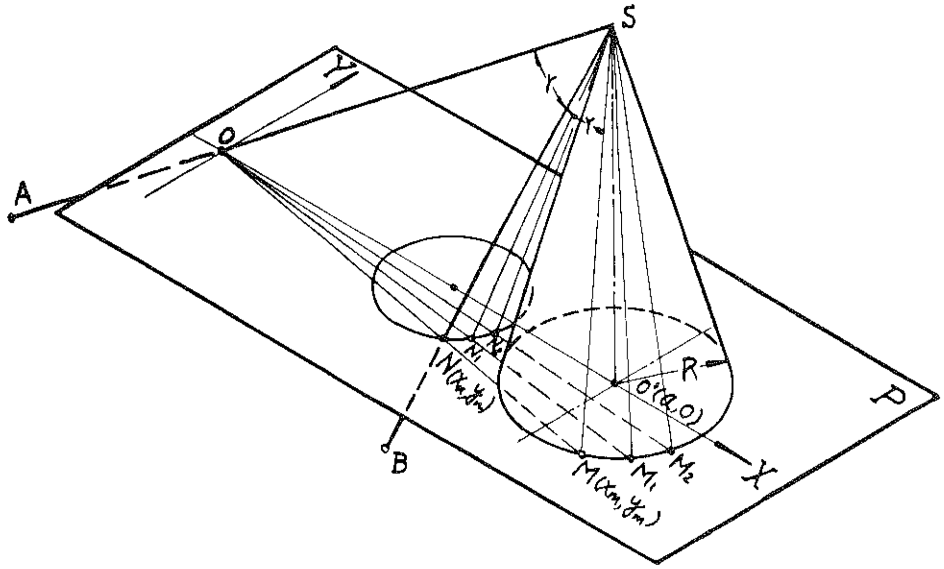


FIGURE FOUR

" $N_1, N_2 \in N$

$M_2, M_2 \in M$

In the triangle OSM, SN_1 is the bisector of the angle OSM_1. Then:

$$\frac{OS}{SM_1} = \frac{ON_1}{N_1M_1}$$

In the triangle OSM_2, SN_2 is the bisector of the angle OSM_2.. then:

$$\frac{OS}{SM_2} = \frac{ON_2}{N_2M_2}$$

Since $SM_1 = SM_2 = SM$, then ...

$$\frac{OS}{SM} = \frac{ON_1}{N_1M_1} = \frac{ON_2}{N_2M_2}$$

by analogy:

$$\frac{OS}{SM} = \frac{ON_1}{N_1M_1} = \frac{ON_2}{N_2M_2} = \ldots = \frac{ON}{NM} = \lambda$$

letting: the coordinate of point M be (X_m, Y_m),
the center be (a,o), and
point N be (X_n, Y_n).

then:

$$(X_m - a)^2 + Y_m{}^2 = R^2$$

$$X_n/(X_m - X_n) = \lambda,$$

$$Y_n/(Y_m - Y_n) = \lambda$$

combining these formulas and simplifying, ...

$$(X_n - a \cdot \frac{\lambda}{1+\lambda})^2 + Y_n{}^2 = (R \cdot \frac{\lambda}{1+\lambda})^2$$

So the locus of the points N is a circle, the radius is

$$R \cdot \frac{\lambda}{1+\lambda},$$

the position of the center is

$$(a \cdot \frac{\lambda}{1+\lambda}, 0)."$$

See 'ya in the fall issue!

Pat

CLEMSON UNIVERSITY OFFERS SEMINARS, WORKSHOPS

Clemson University's division of Continuing Engineering Education (CEE) offers seminars and workshops ideally suited for today's engineering graphics educator. Seminars range from computer programming for graphical displays to computerized design applications.

ANNUAL FALL SEMINAR

Each November since 1977 a seminar on computer automated drafting (CAD) has been offered by the CEE division of the College of Engineering. These two day seminars are extremely well received by the college faculty in the Southeastern United States. The content for the workshop is based upon a textbook entitled "Computer-aided Graphics and Design" which is used in the undergraduate engineering and design graphics course at Clemson University. The emphasis for this introductory seminar is the use of simplified approaches for the computerized procedures that are most useful to the engineer for the creation of drawings. Step-by-step problems and solutions are provided, as well as numerous examples taken from mechanical, electrical, and civil engineering drawing. The scope of the presentation for the seminar is:

1. Detailed explanation of CAD
2. Computer graphics terminology
3. Current application of CAD
4. Manufacturers demonstrations

The content is designed primarily for college teachers and industrial trainers who have need for current updating in this fast moving segment of engineering design graphics. The principle that it is not a replacement for draftsmen, but instead is a tool that they can use to expedite drafting and design assignments -- will be stressed throughout the seminar.

ANNUAL SPRING SEMINAR

Each April or early May since 1978 a seminar on computer programming for graphical displays has been offered. This seminar content is based upon a textbook entitled "Computer Programming for Graphical Displays" which was written for this seminar and published by Brooks/Cole division of Wadsworth. College faculty interested in the content of this seminar will find:

1. Introduction to direct display devices
2. Program instructions/techniques
3. Engineering drawings/documentation
4. Engineering analysis/ animation and simulation techniques.

The content is designed primarily for college teachers or industrial trainers who have large CPU capabilities. FORTRAN and APL type approaches are used throughout. No consideration is given to the use of the BASIC language or other "home" type display devices, only those used in an industrial profit-making situation are introduced.

SUMMER WORKSHOPS

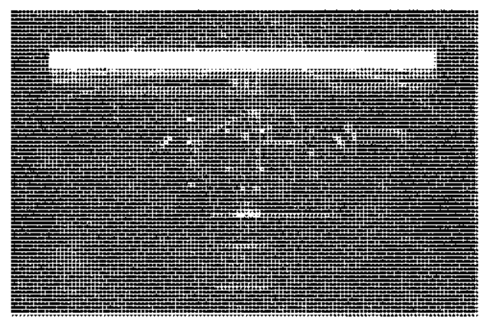
Practical drafting and design criteria along with hands-on operation of computer graphics software are contained in the week long session. Time is set aside during each of the sessions for the laboratory operation. Questions and discussion by the visiting college faculty make this an interesting and worthwhile workshop. An example is the quote by William Taylor, "By 1985, computer graphic output will be used more for mechanical than electrical component design and manufacture."

As far as computer automated drafting is concerned, the means are already here and will be presented during the workshop as:

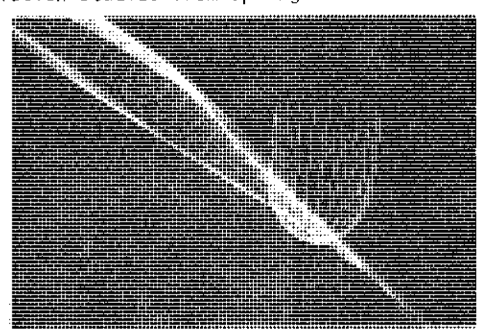
1. Intro to CAD (Computer Automated Drafting)
2. Principles of computer-aided graphics
3. Computer-aided process planning (CAPP)
4. Drafting systems and programming
5. Pictorial representation (CUPID)
6. Computerized descriptive geometry
7. CRT/DVST type terminals
8. Automated vector analysis
9. Computer generated charts and graphs
10. Sample programs and user problems

SEMINAR AND WORKSHOP FACILITIES

The CEE division is located in Rhodes Hall, the engineering research center and newest engineering building housing well-equipped laboratories and classrooms. University and college computer hardware include an IBM 370/3033 system with model 3277 IBM terminal connections to the newest 618 Tektronix "write-thru" DVST. Direct display devices include 10 model 4010's, hard copy units, pen plotters, versatec electrostatic plotter -42", CALCOMP 936, large graphics tablet, and PDP refresh CAD station. The 2200-acre campus is located in the town of Clemson, while another 17,000 acres spred along the shores of Lake Hartwell, approximately midway between Atlanta, Ga., and Charlotte, N.C.



Simulation studies from spring seminar - 1979-80.



Animation studies from workshop/seminar fall 1979.

REGISTRATION INFORMATION

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*Thirty spokes
meet the hub
but it is the
emptiness
between them
that makes
the essence
of the wheel.*

*From clay,
pots are made
but it is the
emptiness
inside that
makes the
essence of
the pot.*

*Walls with
windows and
doors form
the house,
but it is the
emptiness
between
them that
makes the
essence of
the house.*

THE IMMATERIAL IMPARTS ESSENCE.

*The principal:
The material
contains
usefulness
the immaterial
imparts essence.* LAO-TSE

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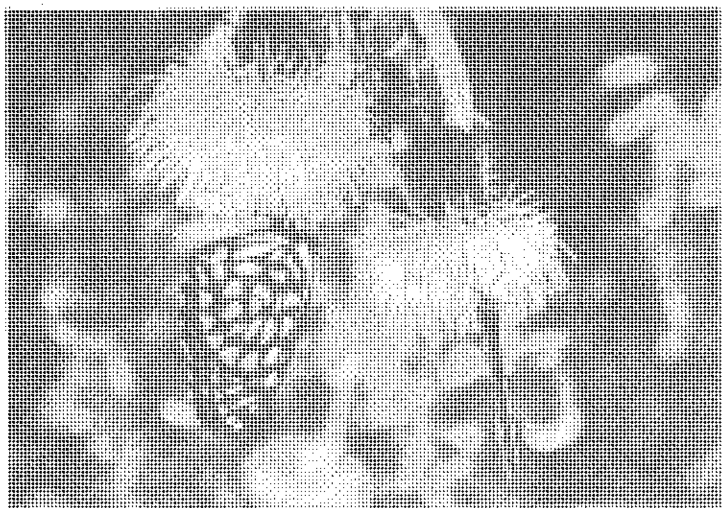

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