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In preparation
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Examination copies of any of these are available upon request.

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Editorial:
Feedback needed

The immense amount of time which your Journal staff devotes to this publication is only repaid by some evidence that you find it lively, useful, educational, or interesting, and that you read it. Send your articles, criticisms, and encouragement to us.

Evidence that some readers at least get as far as the cover was received by your editor shortly after the November '62 issue was mailed. (See Bob La Rue's letter). A.S.E.E. Secretary Collins interpreted the cover as meaning the name of the Division had been changed to "The Division of Engineering Graphics and Design." While this might be a good idea, your editor merely meant the Journal readers to be informed that "Design" was the focus of the issue. This issue is focussed on geometry and analysis in design.

The number of college students seeking an engineering education continues to drop. At the same time over half of those students who do enter engineering studies fail to graduate. Most of the attrition is in the first three semesters and it is in those semesters that engineering graphics is taught. Professor Ralph G. Nevins, Head of the Department of Mechanical Engineering at Kansas State University said, at the 1963 midwinter meeting, "The graphics faculty must be aware of its responsibility to all engineering curricula to conduct their courses in such a way that students are attracted to engineering rather than away from engineering."

Whether the country has too few or too many engineers continues to be a hot debate but is less important than the quality of the practice of engineering in solving manufacturing and research problems of a practical and theoretical nature. The quality of problem solutions is directly related to the visual concepts which underlie the engineer's creativity. Engineering Graphics exercises and develops this ability to conceptualize .

High School Drawing and Advanced Collegiate Graphics

There is increased interest in improving the secondary school courses which would prepare engineering students for advanced graphics in college. Profs. Weidhaas, State Univ. of Penn., Kroner of U. of Mass., and Mochel of Clarkson are working on these projects. On the other end of the spectrum, Profs. Coons of M.I.T., Slaby of Princeton, Levens at U. of Calif., and others are working on Advanced Graphics for engineering research. In the next few years we shall probably see new texts and monographs which will strengthen both ends of graphics education.

Errors

The Nov. 62 Journal scrambled the article by Dr. Mazkewitsch on nomograms. Anyone wishing a corrected article please write to your editor who will send you a copy immediately. This is the fastest way to get you the information as this issue of the Journal has been completed. Your editor regrets the errors and inconvenience. Wanted: - proofreader.

Yours truly,

Mary Blade
Editor

textbooks, manuals, and problem

coming in March, 1963...

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By THOMAS E. FRENCH; and CHARLES J. VIERCK, Ohio State University. Available in March, 1963. A revision of the first major publication to encompass engineering drawing, descriptive geometry, and graphic solutions. As before, the use of uniform terminology throughout the three disciplines has been utilized. Most of the revision has taken place in the descriptive geometry and graphic solutions sections. In these sections the terminology has been simplified and the exposition reworked to bring about a more direct approach; new drawings have been included. Throughout the book the figure captions have been expanded and their pedagogical usefulness increased. The appendix has been expanded to include slide rule material and several new tables.

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Newman A. Hall
Director, Commission on
Engineering Education

Presented at the ASEE Graphics Conference
January 24, 1963

ASEE Graphics Division Midwinter Meeting
Kansas State University

One of the identifying characteristics of engineering activities is the presence and involvement of graphical communication and analysis. This situation has long existed and in spite of changes and evolution of the profession, it will probably persist. Nevertheless, along with other changes that occur one can expect that the role and utilization of graphics will itself evolve in a constructive manner. The direction of change is not always easy to anticipate since it is not immediately involved in those factors which produce these substantial changes in engineering. Graphics tends to play a service role in engineering activity, and, consequently, reflects the influence of other changes but at the same time improvements in graphical techniques can increase significantly the efficiency of engineering activity.

In recent years the rapid developments in engineering and some of the drastic changes in our curricula have made it very difficult for us to maintain a wise perspective when questions arise regarding the role of graphics. Although its place in engineering education has been subordinated we can no more expect that graphics will disappear than we can expect engineering itself to be eliminated from our educational system. It is evident, however, that the nature of graphics instruction and its contribution to engineering activity will be radically different, just as engineering education today has little resemblance to the curricula of 30 or 40 years ago.

We are familiar with the most important factors producing the changes to which we must give attention. Actually, our situation can be described very simply by observing that the rate of change in engineering activity tends to be proportional to the total amount of engineering activity underway at any time. This obviously tends to produce an almost exponential rate of growth so that we can expect that the complexity of our engineering environment will continue to increase very rapidly. In order to keep pace it is necessary for the engineer to continually introduce new scientific knowledge and new techniques in engineering practice and new appreciations of the environment which the engineer will serve. This means that the activity which is underway becomes increasingly complex in view of considerations which enter the picture. At the same time in order for the effort to be maintained this work is no longer going to be undertaken by the individual engineer who will be able to encompass all phases of his responsibility but will rather, be an effort carried on by a team of individuals, each contributing different talents and skills to fulfill the overall engineering objectives. Wise engineering administration will recognize, accordingly, that in its own efforts the most up-to-date techniques must be used by all of the individuals participating and maximum advantage must be taken of the individual skills that the members of the team have at their command.

As a very first step in re-examining the role of graphics we should look carefully at its scope in the engineering environment. The simplest and most fundamental classification indicates that graphics encompasses two aspects. These are communications and analysis. Both are very common in engineering activity, yet we do not always realize that in the largest number of cases these two aspects can be fairly sharply separated. Graphical communication is what most individuals think of as a dominant feature. The engineer who is using a blueprint in some activity is making use of the most common form of graphical communication. The very nature of engineering activity, dealing with tangible objects, systems, and processes makes it inevitable that a variety of types of graphical representations will be useful to convey information about what is under consideration. The need for such communication arises, first of all, within the engineering team itself. At this point, the use of graphics may be very informal, or if a complex activity involving the judgments of many different people, it may be necessary to have a fairly systematic representation of the situation at hand before the final engineering decisions can be completed. In either case, attention needs to be given to the development of those means of graphic communication most directly effective in arriving at the total solution of the engineering problem. The most familiar aspect of graphical communication is that which involves the transmission of information from the engineer to other individuals who will be following his instructions. These may be technicians carrying out test operations or machinists fabricating a part, or others assembling the structure or system which the engineer has specified. The well-established techniques in drafting which have been developed over many years were designed primarily to meet this communication need. These were set up on the basis of the fabrication and assembly techniques which existed and on the methods of engineering analysis which had been worked out. For a long time these procedures have been very satisfactory and in many ways remain so. However, in both regards there are changes underway which not only influence the specific activities of the two groups involved but are consequently effecting the desirable means of communication. Equally familiar to all of us is the need for communication between the engineer and the individual who will be responsible for operation and service of the system or equipment which the engineer has designed. In recent years many very effective means have been developed to present information by pictorial diagrams and photographs in addition to drawings. This is a fairly well developed skill and there are many individuals throughout industry who are adept in providing this sort of information easily and effectively. All of these aspects of graphical communication are ones which must be familiar to the engineer, which he must be able to use readily although in most cases the engineer can rely on the assistance of technicians in the preparation of detailed drawings, charts, or photographs.

The second general aspect of graphics is that of analysis. Although it overlaps communication in some respects, it is quite distinct and of course represents a substantial area in its own right. An individual working in the graphics field in an engineering curriculum will very likely think of graphicsl analysis first in the context of descriptive geometry, which, while very closely related to design, is essentially a means of developing design information by graphical constructions following initial specifications regarding the system under consideration. This particular discipline is representative of a number of graphical constructions which contribute fairly directly to design data and which may require techniques very closely allied to those used by the drafting technician. Another large aspect of graphical analysis consists of the various means of providing graphical solutions of mathematical problems. From an engineering viewpoint it has been very satisfying on many occasions to be able to replace an involved algebraic or numberical analysis of an engineering problem by graphical analysis, since often approximate results are readily obtained and many times there is a direct association with the physical configuration involved in the engineering problem making it easier for the engineer to understand the solution which is being developed. Nevertheless, it is important to recognize that graphical solution to mathematical problems is, first, of all, an aspect of applied mathematics and is no more significantly associated with engineering than is any branch of algebra or the use of a high speed computer. It represents one efficient aspect of mathematical analysis, and if it can be used to advantage, it should be employed by the engineer or any other individual concerned with mathematical studies.

However, it is most important to recognize that there is a whole domain of techniques and procedures which has been introduced in recent years which makes it possible for certain things to be accomplished which formerly were not possible and makes it desirable to introduce practices which are quite different from those which we were familiar with years ago. For example, in conveying information from an engineer to an individual who is responsible for fabrication, there have been developed many short-cuts which change drastically the type of graphical representation which is useful. These include new types of symbolism, the use of photographic techniques, and certainly in this category is the possibility and actual practice of using the computer as a device for preparing graphical information directly. The type of information which must be conveyed has been substantially changed in many cases by the introduction of radically different types of engineering systems and by the introduction of equally different types of manufacturing processes. When this is recognized, it is inevitable that the communication media itself must change its form. Similarly, in the area of graphical analysis it is necessary to realize that as a part of the whole field of mathematical analysis, graphical methods must be used in proper perspective in relation to the improvements in algebraic methods of analysis as well as the use of high speed computers. Situations which were formerly very efficiently handled by graphical methods may no longer be so handled and there are frequent cases on the other hand where combinations of graphical techniques and computers may be used to advantage in ways that were not formerly possible. In general then, one can say that there is as much need for alertness to new developments in graphics as there is in any area which serves engineering education.

These changes which have been outlined briefly suggest that the utilization of graphics in engineering practice will involve three principal characteristics. In the first place, it is certain by-and-large the engineer will need to understand graphical communication as much as he ever has. He will be confronted with this in his work with other engineers and with other individuals with whom he will have to communicate. The ability however, to understand graphical communication is almost inevitably an intrinsic characteristic of an individual who demonstrates engineering talent so that we find many persons who may not have been trained in the skills of graphical communication who are quite able to follow this type of presentation with ease and full understanding. The second feature then of the utilization of graphics is that there will increasingly develop a group of individuals in the total engineering team effort who will be specialists in the techniques of graphical communication. This will include the drafting technician, photographic technician, and others who are similarly specially prepared to make skillful detailed presentation to convey ideas clearly and concisely. The many specialized techniques involved in this practice which are accessible are such that it is most efficient to have a specialist undertake this responsibility rather than to expect the engineer to be able to handle this skill along with his more demanding responsibilities. The third feature is that in the field of graphical analysis there needs to be established a group of specialists -- again as part of the team who are conversant not only with the advantages of graphical methods but are able to use them in a total context of all of the efficient means of carrying out mathematical investigations. Such an individual should know when to use a graphical method rather than a computer or when to combine a computer with graphical methods or how to use a combination of algebraic, graphical, and numerical techniques. The engineer should be familiar with the advantages of the different approaches and in many cases he will be using them to some degree. It will be important for him to appreciate the values of each technique but frequently he will need to call on a specialist who because of his intensive experience can advise the most effective procedure for handling the problem.

What conclusion then, can we draw regarding the direction of development of instruction in graphics? We can identify three broad objectives: First, there should be suitable means for insuring that all engineers are familiar with the methods of graphical communication, as they currently exist, as they are likely to develop, and as they relate to his whole field of engineering knowledge. Second, the engineer should be familiar with and accept graphical analysis as an important tool in any aspect of the analytical investigations which he will undertake as an engineer. Third, there is a very real need for the preparation of a group of individuals as part of the total engineering team who are thoroughly skilled in the techniques of graphical communications.

In looking at these three objectives there is an obvious and immediate division of instructional activity into two categories. There is that instruction which bears on the total education of the engineer, and that which is related to the training of the skilled technician in graphical communication. If the engineer is able to acquire the latter skill he will be able to use it to advantage just as the engineer who is a skilled mechanic can make use of his knowledge effectively. However, the diversity of talents which are required in the total engineering team effort today are of such a nature that it is extravagant to anticipate that all engineers will become skilled in all of the areas supporting their activity.

One of the principal concerns in insuring that the engineer in the course of his education becomes thoroughly familiar both with graphical communication and graphical analysis is the fact that there is a very real tendency to neglect altogether too much these two areas in most of his engineering courses. It is just as important for an engineer to be reasonably effective in graphical communication in almost any of his engineering courses as it is for him to be effective in numerical or algebraic analysis, that is, mathematical communication, or in use of the written word. Any means that can be developed to emphasize the importance of this phase of engineering activity and to insure that the graphical techniques will be used in appropriate balance in all engineering courses will do more to place graphics in proper perspective in the educational program than anything else. Such efforts will be far more effective in insuring that there will be a well balanced demand for basic courses in graphics than any defensive action which might be set up with regard to such courses. A major reason why graphics courses have come into disfavor in many cases is that there has not been maintained evidence of the continual usefulness of the knowledge imparted by such courses either in engineering practice or in advance courses in engineering. If the engineering faculty as a whole does not make use of graphical communication and does not require its use in courses there will be certainly little defense or need for fundamental courses in these areas. Similarly while graphical analysis can be a very powerful tool, it should not be divorced from other aspects of mathematical analysis. An individual who is concerned with its effective use must be in a position to insure that its use will be discriminating along with the appropriate use of algebraic, numerical, and computational procedures. We must not forget that a course in graphical analysis is basically a course in applied mathematics and should be judged on this basis. One of the most effective means of keeping graphics in its proper position is to develop a continuous flow of up-to-date information which demonstrates the value, relevance and usefulness of graphics in all phases of engineering instruction and activity. If this is done effectively so that the entire engineering faculty can be conversant with the value and possibilities of graphics as it relates to their activity, then a proper balance will develop in a very smooth manner.

Those of us who are concerned, who have the maintenance of skills in graphical communication, should be giving a great deal of attention to the instruction of individuals whose primary contribution to the engineering team effort will be along this line. This can be a significant contribution and one that can be very stimulating in its own right. Certainly in the appropriate place in the total educational effort, there must be a pattern of courses which will make possible the training of such specialists. Such individuals must be thoroughly familiar with established drafting techniques, with some of the most up-to-date equipment and procedures, probably with techniques in photographic communication and increasingly with the use of the computer in graphical communication. Here again the problem which confronts our faculties concerned with this area is that we do not have sufficient current information regarding procedures and practices which are becoming well established in industry so that we can organize and maintain courses and programs of instruction which provide a high level of motivation for the appropriate students. Any effort that can be carried forward to insure that such information is provided in a readily usable form as well as procedures which will make it possible for our faculties to become thoroughly familiar with the use of the procedures will be of greatest value.

In conclusion then, we can observe that graphics is a substantial aspect of engineering activity. The engineer will continue to make this a very real part of his total activity. His education must make it possible for him to use graphical methods easily and effectively. At the same time the total engineering team must also include individuals who are specialists in one part or another of the field of graphics.

GET THE POINT

Richard W. Gohman, Instructor,
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The following monograph is concerned with proving a relationship between specific connectors of two skew lines. Use of this relationship can be made to save considerable projection space and time when more than two of the specified connectors are desired.

It has been observed that in the elevation view where two skew lines appear parallel, the shortest connector, the shortest horizontal connector, the shortest grade connectors, and the vertical connector show in true length and appear to cross at a common point. This is illustrated in the attached figure. If this apparent crossing point always exists and is singular in nature, then once located by two of the specified connectors, it becomes a convenient locator for any of the remaining connectors under discussion, thus eliminating the otherwise necessary point projection view. The elimination of any projection view in a graphical problem solution must result in both a savings of projection space and time.

It is the purpose, then, of this discussion to prove that such an apparent intersection or crossing point will always exist and exist at only one point within a given projection.

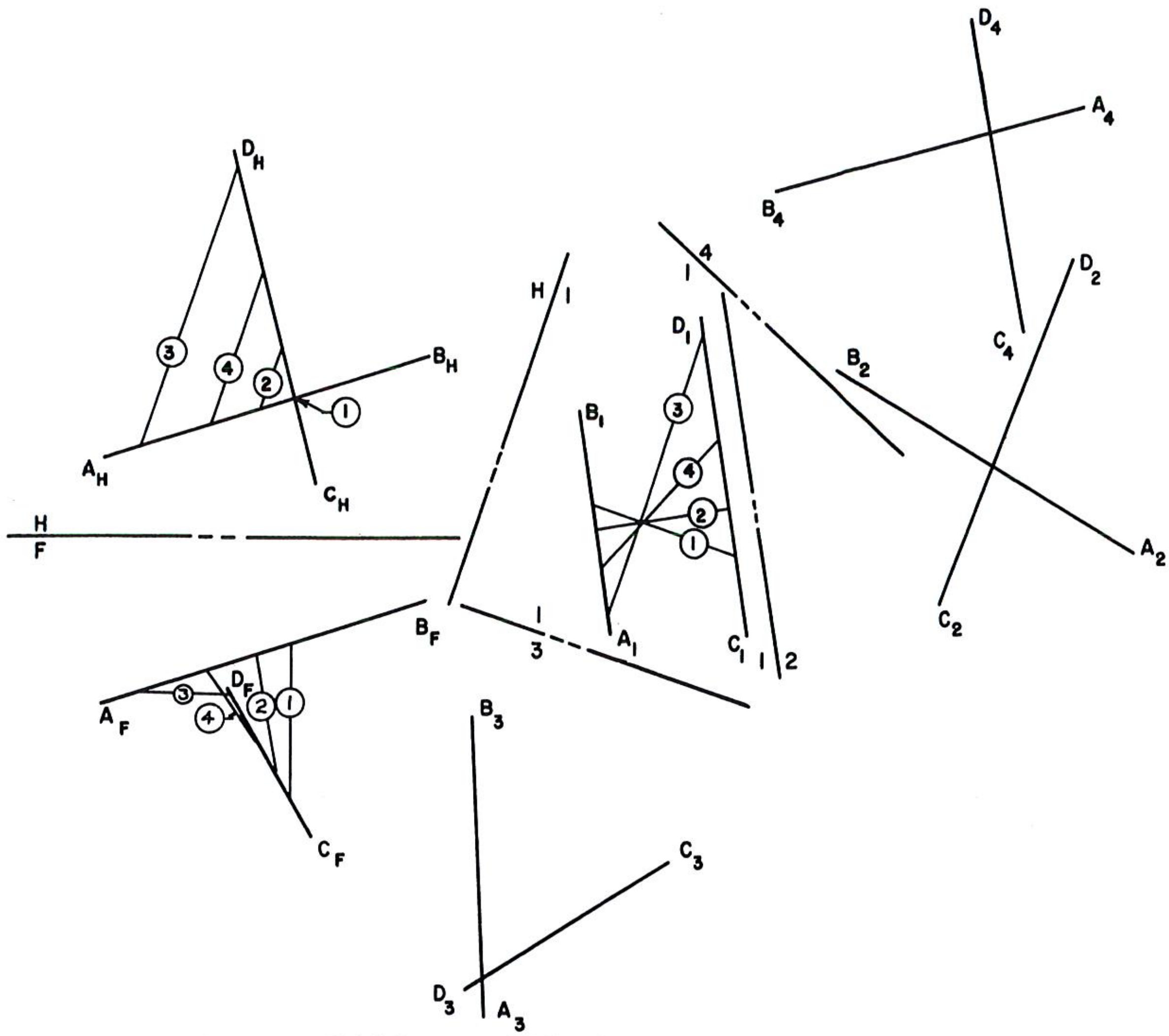
First, consider the fact that if the vertical connector, the shortest horizontal connector, the shortest grade connectors, and the shortest connector all show true length in the same view, they are parallel to the same plane, namely the viewing plane. Such straight lines parallel to a plane and connecting two skew lines are by definition elements of a hyperbolic paraboloid. As such, it should be noted that these connectors are nonparallel and nonintersecting. Consider also that the two skew lines are parallel to a plane and connect any two of the aforementioned connectors. Hence, also by definition, the skew lines are two elements of an additional set of rulings in the same hyperbolic paraboloid. The hyperbolic paraboloid is therefore double ruled.

Next consider that in a hyperbolic paraboloid, all the elements directed by a plane director appear parallel when the plan director is projected as an edge. But, since by definition these elements are not parallel, they must each form a different angular value with the viewing projection. Then in a full set of elements, one and only one can form a right angle to the view and be projected as a point. When in addition ion, the hyperbolic paraboloid is double ruled, the single element of one set of rulings appearing as a point must also intersect each element of the remaining set of rulings; thus the remaining set of rulings will appear to intersect each other at a common point.

Lastly, consider that if the two skew lines are viewed in the elevation view where they appear parallel, the conditions detailed above for a hyperbolic paraboloid do apply. The plane director for the set of rulings that the skew lines belong, appears as an edge and the hyperbolic paraboloid has been shown to be double ruled. Therefore, a single element from the set of rulings that includes the skew lines must appear as a point. This same element must also intersect the remaining set of rulings, namely the aforementioned connectors; thus these connectors appear to cross each other at a single point.

The proof is now established that when two skew lines are projected to appear parallel in an elevation view, the shortest connector, the shortest horizontal connector, the shortest grade connectors, and the vertical connector will appear to cross at a single point. This point, once established, may be used to locate any additional connectors of this specific group without additional views.

See Diagram →



CONNECTORS BETWEEN SKEW LINES

(1) VERTICAL CONNECTOR
(2) SHORTEST CONNECTOR
(3) SHORTEST HORIZ. CONNECTOR
(4) SHORTEST 43% GRADE CONNECTOR

IMPROVING EXAMINATIONS BY STATISTICAL ANALYSIS

By Dr. Robert P. Borri
Associate Professor of General Engineering
University of Illinois

Introduction. Because of the impact of modern science on engineering education, there has been in recent years much activity in the reorganization of curricula and in the re-evaluation of degree requirements. In many quarters, there is a prevailing notion that these are the cure-alls of most engineering educational problems. We somehow tend to forget that the educational process has many facets and that the curriculum is only an educational instrument which derives its integrity, strength, and effect from the capacity of the teacher to instruct and inspire. I believe that there is a need for a balanced approach for improving all facets of the engineering education process. Therefore, it is the purpose of this paper to focus some attention on the basic principles underlying one of the instructor's chief devices, the examination. While the examples cited in this report deal with tests in engineering graphics, many of the principles apply equally well to testing in all areas of engineering.

Purposes of Examinations. One use of examinations is to measure a student's achievement in a subject field and may be thought of as a "feed-back" device for keeping the student "on course." Another use is to diagnose a student's strengths and weaknesses and, in general, to show his proficiency in the subject. In addition, an examination is often used as a teaching device and as a basis for determining a student's grade for a course.

Characteristics of a Good Examination. Validity is the degree to which a test measures what it is supposed to measure--that is, the test items should be fundamental and in accordance with the objectives and contents of the course.

Reliability refers to the consistency of various parts of the test. The method of finding the coefficient of reliability of a final examination by the "split-half" technique is shown in Table 2. The data used in this table are from an examination having items involving 185 points and include the scores of eighty-seven students. The test was divided into two parts--the odd-numbered parts were compared with the even-numbered questions--and the degree of consistency between the two parts was compared. The coefficient of reliability for this particular test was found to be .93 for either half of the examination. By using the Spearman-Brown prophecy formula, the estimated reliability for the entire examination was found to be .96. Since "1" means perfect correlation, .96 indicates a very satisfactory rating.

Achievement should not only determine a student's grade or rank in the class, but it should also indicate the difference between acceptable and non-acceptable performance. In addition, it should point out where the student's performance can be improved. An important outgrowth of testing should be the establishment of standards of mastery.

Objectivity in a test refers to removing the personal factor of the person scoring the test and, in addition, all students knowing the information should get the same meaning from the same test item. Comprehensiveness re-refers to adequacy of sampling in relation to available time. Ease of administrating and scoring also are other desirable attributes of a good test.

Types of Examination Questions. In most subjects, the instructor has several types of questions available for the construction of a test. Some of the more common types are: essay, problem, performance, and objective-type questions such as: true-false, multiple-choice, completion, and matching. Each type of question has certain advantages as well as certain limitations, and the subject matter, objectives of the course, and time available are factors that determine the types of questions that are most suitable. Framing good questions is an asset to good teaching. A performance part of a test has the advantage of checking a student's ability to apply the knowledge he has acquired. An essay-type question usually requires students to organize their knowledge and demands an ability to express ideas; increasing the number of questions and reducing the amount of discussion on each by using a suggested time limit are desirable. Many test-makers agree that the multiple choice test can be effectively used in a wider variety of situations than any other test.

Calculating Test Scores. The data in Figures 1-4 are taken from a Graph and Charts tests formerly used in a beginning course in engineering graphics. It consisted of twenty questions and shows three different ways of calibrating the test results. Figure 1 shows the grade distribution made by setting an arbitrary number of point credits before giving the examination and then plotting the curve into percent values. Figure 2 is based on sixty-one points, also, but the places where the grade breaks were eventually made were not determined until after the results of the test were plotted. Figure 3 shows the distribution curve based on a plan of allowing a value of 5% for each of the twenty questions and converting to letter grades according to certain per cent equivalents. Figure 4 shows a comparison of the achievements in this quiz for classes of three different years.

By inspection of Figure 3, one notes that the grades vary from 100% to 35%, a range of 65%. The mode, the frequency with the largest number, is 95%, and the arithmetic mean or average is 85.6%. The standard deviation (σ), the most reliable measure of variability, is found to be 12.25% as shown in Table 1.

Methods of Expressing Test Scores. Test scores of students acquire meaning when compared with scores or norms of well-identified groups of individuals. Every textbook on testing discusses the characteristics of the bell-shaped curve; many of the methods of deriving meaningful scores are based on the properties of this curve, Figure 5. The total area under the curve represents the total number of scores in the distribution. One important observation is that about two-thirds of all cases lie between plus and minus one sigma. Another worthwhile relationship is that 1σ is equivalent to the 16th percentile, 0 to the 50th percentile, and - 1o to the 84th percentile. This is true, of course, if the group closely approximates the theoretical normal distribution.

By visual inspection of Figure 3, we note that the distribution of grades obtained on this test does not closely resemble the normal curve, but instead resembles a truncated normal curve. By using the same average of 85.6% and the same standard deviation of 12.25% as on actual distribution, Table 3 normalizes the distribution;

We note that where a normal curve has three or four standard deviations on either side of the mean or average, this disbribution has only one standard deviation above the average but has three standard deviations below the average. One precise method of comparing the obtained distribution with the normal distribution fitted to the same data is Pearson's Chi Square Test for goodness of fit, Table 4. The large value of Chi Square (232.6) is a mathematical corroboration of our previous observation; a graphical comparison is shown in Figure 6. Both methods verify our visual inspection that the actual disbribution of these test grades do not follow the normal curve; this is due in part to the size and selectiveness of the group taking the quiz.

Computing Difficulty Indexes of Items. Many ways of expressing the difficulty level of an item have been proposed. The most common of these is to calculate the per cent of the group that answers the item correctly, and this is known as the difficulty index. In theory, this index may vary from 100% for items which every one answers correctly to 0% for items which no one gets correct. The index of items will actually fall in between these two extremes, with the desired situation calling for most items of the 50% level.

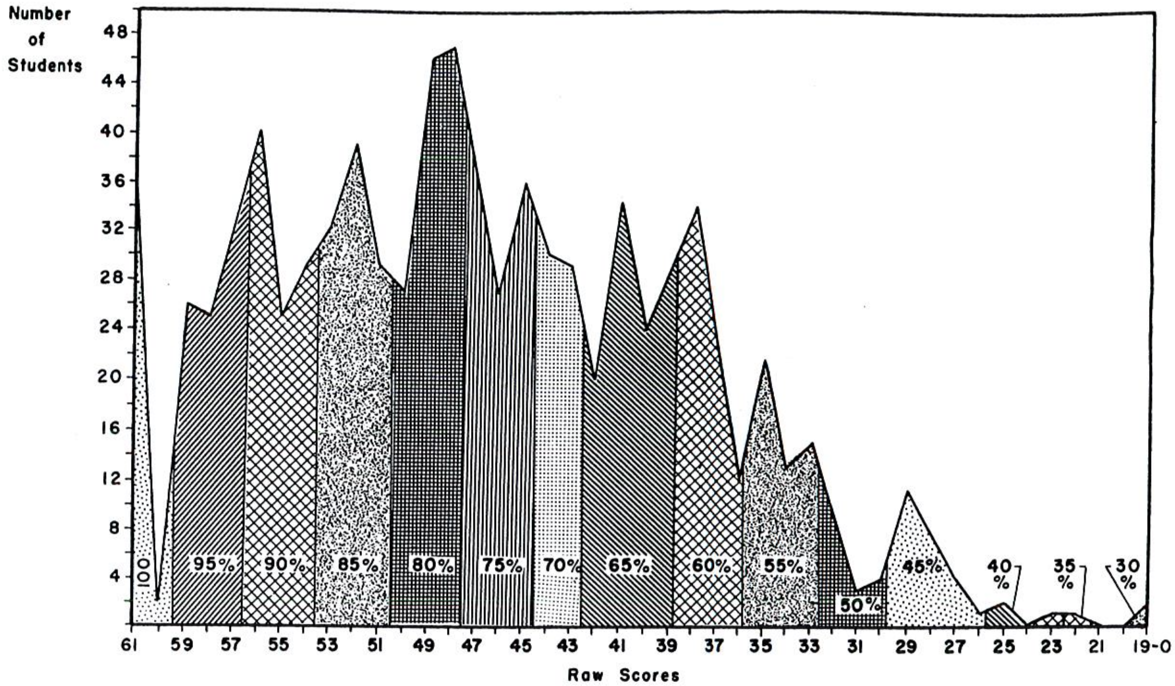
Some items above 50% and some below are normally included in a test in an attempt to motivate the poorer students and to challenge the better ones. Of course, some items with extremely low or high indexes may also be included because of the fundamental nature of the items, making it desirable that all students know them. Table 5 shows the difficulty index for each item in the Graphs and Charts Quiz.

Computing Discrimination Indexes of Items. If an item is answered correctly by all of the good students and is missed by all of the poor students, we consider that item as having perfect validity (1.0); however, such items are rare. Most items will fall along a scale ranging from no discrimination to highly valid percentages.

Over fifty different statistical procedures have been devised to show the degree of discrimination for various test items. Some involve the use of intricate formulas, while others are simple and may be used profitably, with little labor, by any instructor.

continued on following page

Figure 1



Grades in Percent on Graphs and Diagrams Quiz in G.E. 101 with Calibration based on Mathematical Formula (845 Students)

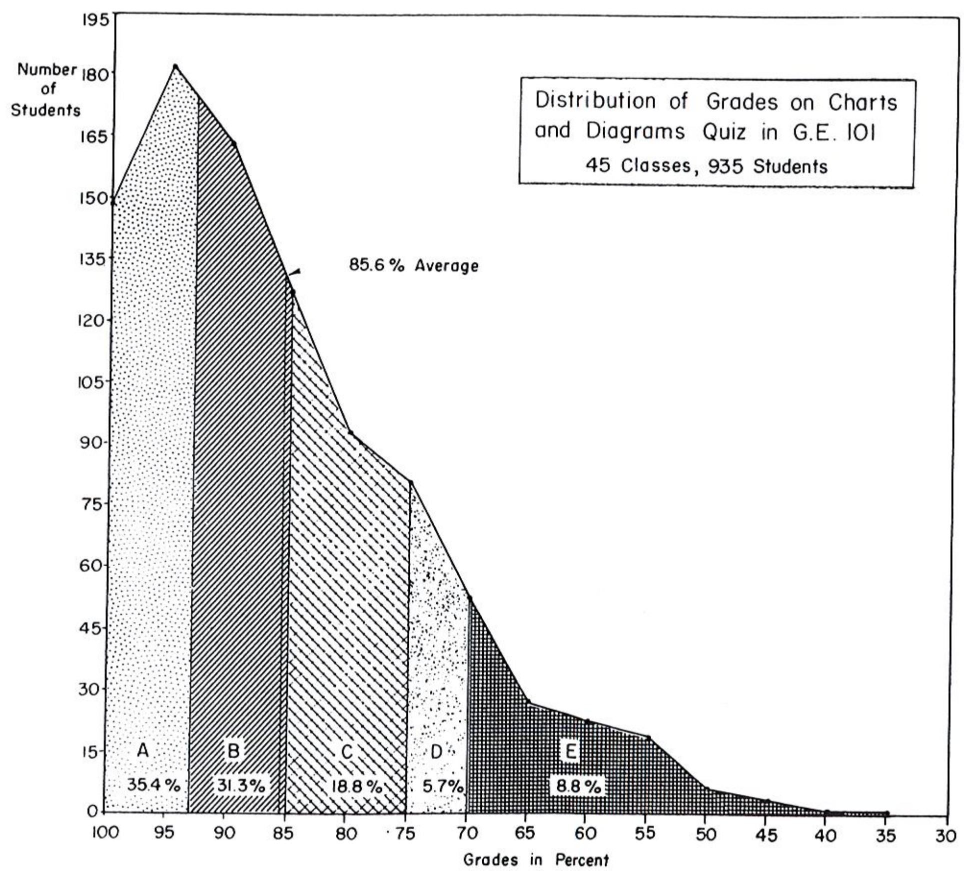


Figure 3

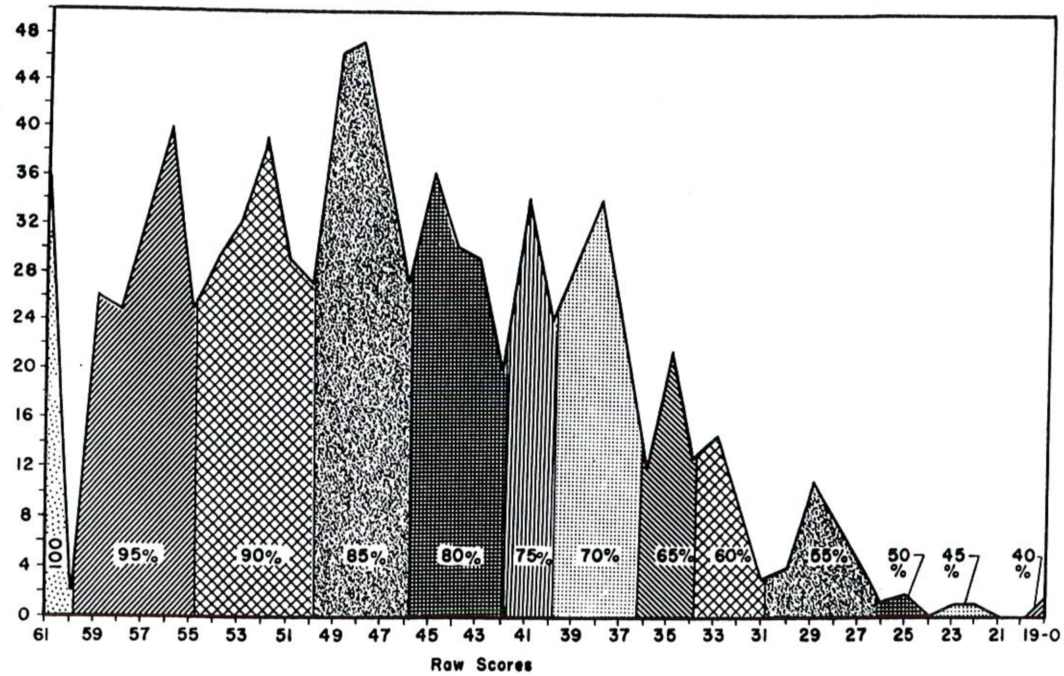
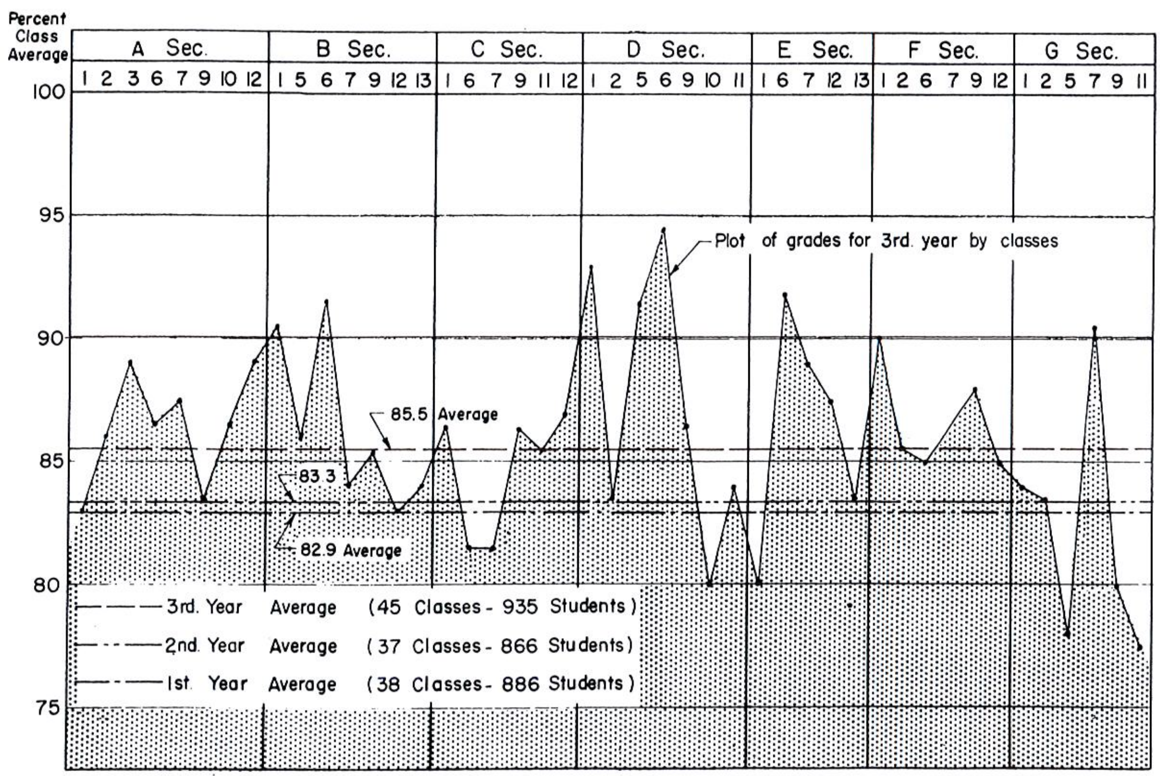


Figure 2

Grades in Percent on Graphs and Diagrams Quiz in G.E. 101 with Calibration based on Achievement of 845 students



Average Grades on Charts and Diagrams Quiz for Three Years

Figure 4

TABLE 1.

Computation of Mean and Standard Deviation of the Frequency Distribution of the Grades Made by 935 Students in Charts and Diagram Quiz in Engineering Graphics

Grades X	Students f	Xf	$(X-\bar{X})$ x	x^2	fx^2
100	147	14,700	14.38	206.784	30,397.248
95	184	17,480	9.38	87.984	16,189.056
90	163	14,670	4.38	19.184	3,126.992
85	130	11,050	- .62	3.844	499.720
80	93	7,440	- 5.62	31.584	2,937.312
75	83	6,225	-10.62	112.784	9,361.072
70	53	3,710	-15.62	243.984	12,931.152
65	27	1,755	-20.62	425.184	11,479.968
60	23	1,380	-25.62	656.384	15,096.832
55	19	1,045	-30.62	937.584	17,814.096
50	6	300	-35.62	1,268.784	7,612.704
45	5	225	-40.62	1,649.984	8,249.920
40	1	40	-45.62	2,081.184	2,081.184
35	1	35	-50.62	2,562.384	2,562.384
Total	N=935	80,055	...	10,287.636	140,339.641

Arithmetic Mean $(\bar{X}) = \frac{\Sigma Xf}{N} = \frac{80,055}{935} = 85.62\%$

Standard Deviation $(\sigma) = \sqrt{\frac{\Sigma fx^2}{N}} = \sqrt{\frac{140,339.384}{935}} = 12.25\%$

TABLE 2.

Calculation of the Reliability Coefficient of the G.E. 101 Final Examination. Method of Chance Halves.

Pupil	X Odds	Y Evens	x'	y'	x'^2	y'^2	$x'y'$	Pupil	X Odds	Y Evens	x'	y'	x'^2	y'^2	$x'y'$
1	85	86	5	8	25	64	40	46	83	84	3	6	9	36	18
2	88	87	8	9	64	81	72	47	85	81	5	3	25	9	15
44	86	87	6	9	36	81	54	86	85	79	5	1	25	1	5
45	87	83	7	5	49	25	35	87	81	82	1	4	1	16	4
(Sums)											-124	-122	3964	4300	3857
Guessed Mean									80	78					

$$r = \frac{\frac{\Sigma x'y'}{N} - \left(\frac{\Sigma x'}{N} \cdot \frac{\Sigma y'}{N}\right)}{\sqrt{\frac{\Sigma (x')^2}{N} - \left(\frac{\Sigma x'}{N}\right)^2} \cdot \sqrt{\frac{\Sigma (y')^2}{N} - \left(\frac{\Sigma y'}{N}\right)^2}} = \frac{\frac{3857}{87} - \left(\frac{-124}{87} \cdot \frac{-122}{87}\right)}{\sqrt{\frac{3964}{87} - \left(\frac{-124}{87}\right)^2} \cdot \sqrt{\frac{4300}{87} - \left(\frac{-122}{87}\right)^2}} = .9313$$

$r = .9313$, reliability coefficient for either half of the examination. Estimated "r" of the Entire Examination by use of the Spearman-Brown prophecy formula: $r_{nn} = \frac{nr}{1+(n-1)r} = \frac{2(.9313)}{1+(2-1).9313}$ $r_{nn} = .964$

In a case where nearly every student has had sufficient time to answer each item and where the samples are small, and involve no assumptions regarding the shape of the distribution of the items measured, the following simple method is applicable. One selects the highest 27% and the lowest 27% of the test papers, being sure to have the same number of papers in each group. (This number is used because with normal distributions, 27% is the point at which one gets the maximum differences with maximum reliability.) The correct answers to each of both groups are tabulated, and a percentage figure showing the relationship between the two groups is calculated. Item discrimination indexes of above .25 are considered that have little discrimination, if the item is considered basic and important enough that every satisfactory. However, a test may include items student should know it. It is also possible to have items that discriminate negatively--that is, items to which the poor group could get more correct answers than would the good group. In such a situation, such items might be saved by overhauling them. The discrimination indexes of each of the twenty items in the Graphs and Charts Quiz is shown in Table 6.

Graduation of Data of Charts and Diagram Quiz to Normal Distribution

$\bar{X}$ = 85.62% . σ = 12.25% . y = (382.26) z.

Grades X	$(X-\bar{X})$ x	$\frac{X-\bar{X}}{\sigma}$	Table Values z	Theoretical Frequency y
100	14.38	1.17	.2012	76.02
95	9.38	.77	.2966	113.38
90	4.38	.36	.3739	142.93
85	- .62	.05	.3984	152.29
80	- 5.62	.46	.3589	137.19
75	-10.62	.87	.2732	104.43
70	-15.62	1.28	.1758	67.20
65	-20.62	1.68	.0973	37.19
60	-25.62	2.09	.0449	17.16
55	-30.62	2.50	.0175	6.69
50	-35.62	2.91	.0058	2.21
45	-40.62	3.32	.0016	.61
40	-45.62	3.73	.0004	.15
35	-50.62	4.14	.0001	.04

Ordinate (y) $= \frac{N}{\sigma} z = \frac{935}{2.446} z = (382.26) z$

TABLE 3

Determining the Effectiveness of Misleads. The efficiency of a multiple-choice item is the quality of its misleads or incorrect alternatives. If they are illogical, absurd or in any way inattractive, the items are not valid, because the student has no difficulty in answering the question. It is important to the test-maker that he know to what extent the poor and good students choose the various answers within each item. The procedure for finding the effectiveness of misleads is the same as that used for determining item discrimination, previously covered and shown in Table 7. We see from this table that mislead "C" needs improvement; it is very ineffective since only two students out of 80 chose it.

Recording Item Data. The three kinds of item data described previously can all be obtained from the same source. This information can be recorded on a 5 by 8 index card and filed as a part of the cumulative item pool, which might include heading entries such as: "Course and Unit," for filing purposes; "Objectives," to get full and evenly distributed coverage; "Item," as it appears in the quiz; "test Form," which identifies the location of the item in each test form; and "Administration," which shows the distribution of responses for the upper and lower 27% of each class taking the test. Data on new items may be found by systematically including several items with each test form; the students are not graded on the new items, but the items are evaluated and a file of new items of known value can be developed in a short time.

Computation of Chi-Square Test to Check Normality of Data on Charts and Diagram Quiz

Grades X	Actual f	Theoretical fc	f-fc	$(f\text{-}fc)^2$	$\frac{(f\text{-}fc)^2}{fc}$
100	147	76.02	71.0	5,041.00	66.31
95	184	113.38	70.6	4,984.36	43.97
90	163	142.93	20.1	404.01	2.83
85	130	152.29	-22.3	497.29	3.27
80	93	137.19	-44.2	1,953.64	14.24
75	83	104.43	-21.5	462.25	4.43
70	53	67.20	-14.2	201.64	3.00
65	27	37.19	-10.2	104.04	.36
60	23	17.16	5.8	33.64	1.96
55	19	6.69	12.3	151.29	22.61
50	6	2.21	3.8	14.44	6.53
45	5	.61	4.3	18.49	28.02
40	1	.15	.8	.64	3.76
35	1	.04	.9	.81	31.33
Total	935	...	...	...	232.62

TABLE 4

$$\text{Chi Square } (X^2) = \sum \frac{(f\text{-}fc)^2}{fc} = 232.62$$

Table 5

Calculation of Difficulty Indexes from Results of 250 Students on 16 Items of graphs and Charts Quiz

Item No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
No. Correct	236	195	182	185	223	217	195	238	186	123	102	102	242	195	214	180
% Correct	94.4	88.0	72.8	74.0	89.2	86.8	88.0	95.2	74.4	49.2	40.8	40.8	96.8	88.0	85.6	72.0

Table 6

Calculation of Discrimination Indexes from Top and Bottom 27% Groups of 150 Students, Graphs and Charts Quiz

Item No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
High 27%	40	36	35	37	38	38	38	40	37	32	28	29	40	39	37	38	35	40	40	40
Low 27%	35	30	19	23	34	32	27	34	18	10	9	7	39	26	30	20	20	36	23	35
Index %	.13	.15	.40	.35	.10	.15	.27	.15	.47	.55	.47	.55	.03	.33	.17	.45	.37	.10	.43	.13

Table 7

Number of Students in Each Group Who Chose Each Alternative, Graphs and Charts Quiz

Item No. 10	A	B	C	D*	Omissions	Total
High 27%	3	4	0	33	0	40
Low 27%	5	23	2	10	0	40

* = correct answer

Table 5

Figure 5 The Normal Curve

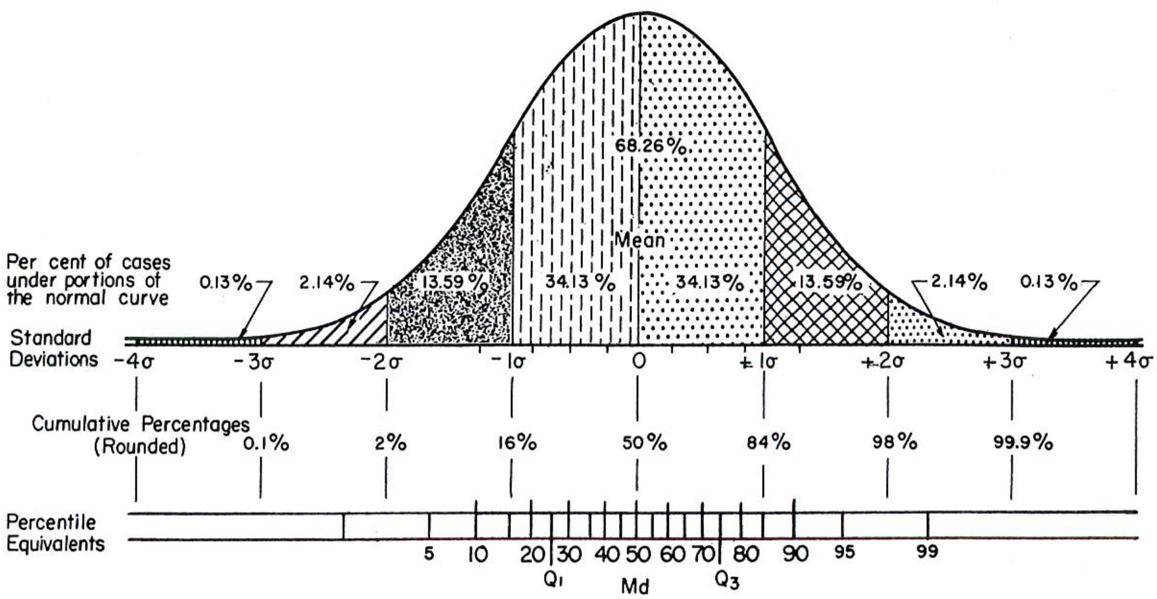


Figure 5

Figure 6.

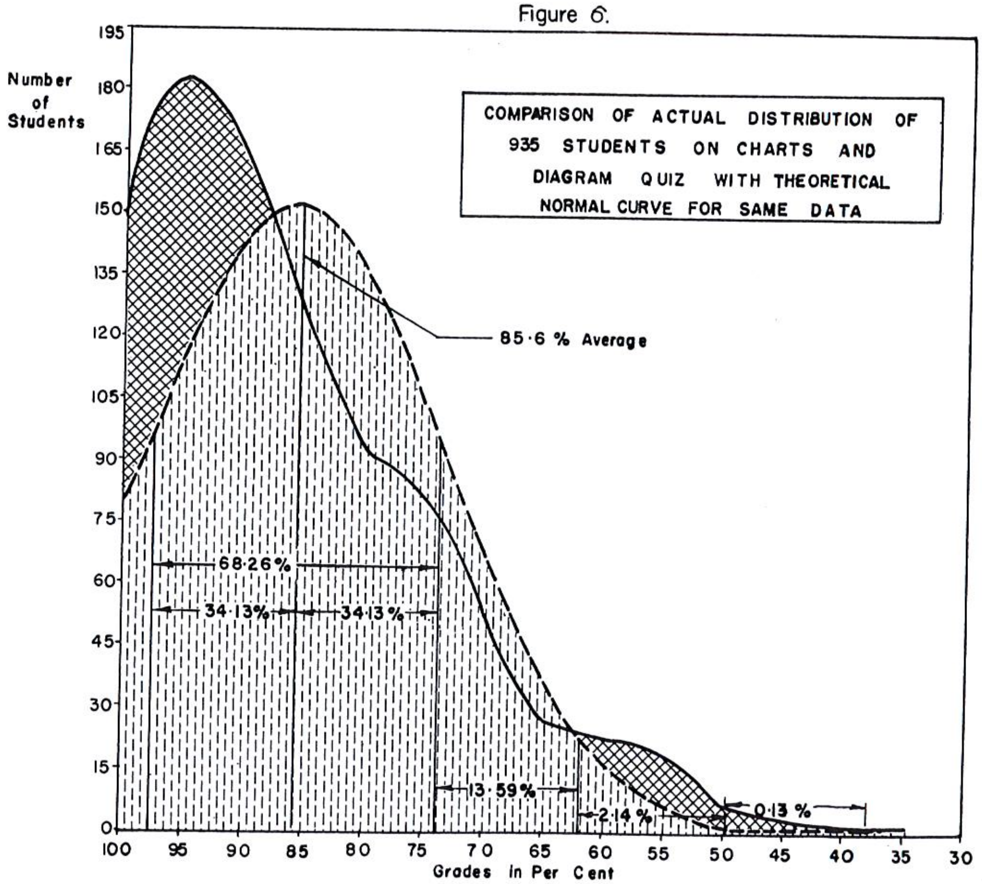


Figure 6

CONCLUSIONS:

1. Since the examination is one of the instructor's chief devices and is an integral part of every well-organized course of instruction, it is important that improvement in examinations keep pace with the recent science-motivated changes in the engineering curricula.

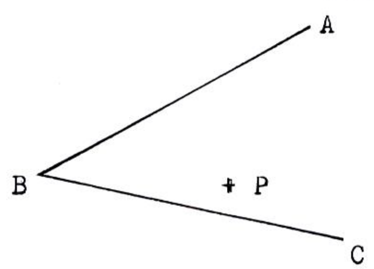
2. Examinations may be used for various purposes, the chief ones being: to keep a student "on course," to diagnose a student's strengths and weaknesses, to show his proficiency in a subject, and to serve as a basis for determining a students' grade in a course.

3. A good examination has several characteristics which a test-maker should strive to incorporate in the test; namely, that it be valid, reliable, objective, and easily administered and scored.

4. A statistical analysis of test results involves finding the difficulty and discrimination indexes of test items, and is a means of improving the examination, the course of study, and the efficiency of instruction.

GRAPHICS TANTALIZER

by Dr. Andrew Longacre
L.C. Smith College of Engineering
Syracuse University



Locate the shortest connector between intersecting lines AB and BC, that passes through an arbitrary point P, within the angle ABC.

GRAPHIC TANTALIZER

Given three skew lines. Find a plane which makes a given angle x with the three skew lines.

ADMINISTRATION OF AN ENGINEERING GRAPHICS AND DESIGN COURSE - One Way To Do It.

A. S. Levens

Editor's note: Readers of the Journal have been asking for articles on how various university courses in graphics are administered. Professor Fernandez of Manhattan College started the series a year ago. We now are fortunate in having Professor Levens give us an article on the administration and the course content of the Engineering Graphics and Conceptual Design program of the University of California at Berekeley.

At the outset, it should be pointed out that the courses in graphics - (a) Engineering Graphics and Conceptual Design; (b) Nomography; and (c) Graphical Analysis - are part of the offerings in the Mechanical Engineering Department which is composed of four divisions: Heat Power; Aeronautical Sciences; Applied Mechanics; and Mechanical Design. The latter includes Engineering Graphics and Conceptual Design; and Statics (at the freshman-sophomore level); Dynamics; Mechanism and Dynamics of Machinery; Machine Design; Design of Mechanical Equipment; Advanced Mechanics; Individual Study or Research for Advanced Undergraduates; and Nomography at the junior-senior level; and Advanced Kinematics and Mechanisms; Optimal Design of Mechanical Elements; Lubrication and Friction; Graphical Analysis; Group Studies; Seminars or Group Research; and Individual Study or Research -- at the Graduate level.

Senior faculty members of the Mechanical Design division are in charge of courses that have multiple sections. The writer is in charge of the graphics area.

Now with regard to the latter the operation of the required course, "Graphics and Conceptual Design," is as follows:

(a) In the fall semester of the freshman year there are 16 or more sections of approximately 25-30 students each.

(b) The students attend two one-hour lectures, one three-hour laboratory period; and do an average of four hours of homework per week.

(c) There are three lecture groups of approximately 175, 150, and 100 students respectively.

(d) The largest lecture group meets on Mondays and Fridays at 1 pm and the second largest group on Tuesdays and Thursdays at 1 pm. The first half hour of the Monday lecture is video-taped and played back via T.V. monitors, to the Tuesday lecture group. On Thursdays, the first half hour of the lecture is video taped and played back to the Friday lecture group. The second half of each lecture period is devoted to a "live" question-and-answer period.

The third lecture group meets on Tuesday and Thursday mornings at 8 AM. In this group there is no video-taping.

Last fall semester was the first time we used the method described above. The results have been quite satisfactory. There are several advantages in using the combination of TV with "live" discussion when scheduling, such as mentioned above, permits.

In the first place, lectures must be very carefully prepared (there is nothing like a playback that shows you in action before your students). Secondly, it affords a fine opportunity for the less experienced members of the staff who conduct the laboratory sections to observe good teaching techniques and to gain knowledge of the material that is "uncovered" at each lecture. This results in a good correlation between the lecture material and the laboratory work. Moreover, it also affords an opportunity for the laboratory staff to participate in the discussion portion of the lectures.

Most of the laboratory instructors are graduate students who have had little or no teaching experience. If we are to develop competent engineering teachers to meet present and future needs, it is incumbent upon us, who are experienced, to make a genuine effort to help these young men.

Staff seminars and individual conferences, although time consuming, are essential to the effectiveness of "teaching internships" that can help produce potentially fine teachers of engineering.

A very important role of the laboratory staff will be discussed shortly in connection with course content.

During the first 6 1/2 weeks of the course stress is laid on the fundamental principles of orthogonal projection and their application to the analysis and solution of three-dimensional problems that arise in the various fields of engineering. I should preface this statement by pointing out that in the initial lecture - first meeting with the students - it is essential to discuss (a) the engineering profession - what it is; (b) what is science; (c) what are the salient differences between science and engineering; (d) the importance of mathematics, physics, chemistry, graphics, the engineering sciences, and the humanities and social sciences to the engineer; and (e) the specific role of engineering graphics and conceptual design in engineering education.

Setting the stage and stimulating the students' interest in engineering at the very outset, is a must! Once this is done I would suggest that the follow up be a preview of the course content.

The next 3 1/2 weeks are devoted to graphical calculus; empirical equations; functional scales; and an introduction to nomography. The final 5 weeks are devoted to conceptual design.

I believe, strongly, that modern curricula in engineering must lay greater stress on synthesis. There is reasonably good agreement among engineering educators that design is the core of engineering. Design, in its broadest sense, includes circuits, processes, structures, and machines. What is lacking in present-day engineering curricula, in my humble opinion, is the opportunity for engineering students to synthesize the background material which they have achieved at each level of their education. At the freshman level a course such as Engineering Graphics and Conceptual Design provides a means for integrating the fundamentals of projection theory; freehand expression; graphic mathematics; physics; chemistry; and English. "Open-ended" projects are admirably suited to the implementation of this integration through the conceptual design phase of such a course. Present engineering curricula lay major stress on analysis. Since synthesis precedes analysis it is essential that our students gain early experiences in design at several levels - freshman, sophomore, junior, senior, and graduate. The freshman year is not too early to introduce conceptual design!

Characteristic of the design problem is that there is no unique solution. A number of adequate answers is possible, some of which may be better than others.

The creative art of conceiving a physical means of achieving an objective is the essential and most crucial step in an engineering project. The conceptual design process (synthesis) is the important first step; analysis, the important second step. Through the employment of "open-ended" problems most freshman engineering students can express their creative abilities, quite successfully in conceptual design. The students' background, in mathematics, engineering graphics, chemistry, English, etc., can be employed in creating solutions to "real technological problems" appropriate to the students' level of preparation. A freshman engineering course such as engineering graphics and conceptual design, integrates the above mentioned areas quite well, and contributes, significantly to stimulating the students' interest in engineering.

Typical of "open-ended" projects that have been used in our course are the following:

1. Design a small toy automobile which will never fall from an edge but will turn at least 90° as soon as the front wheels begin to drop after crossing the edge. The toy should not exceed four to five ounces, should be set in motion by a spring acting on the rear wheels, and should operate on a 1 1/2' x 1 1/2' plywood board for a minimum of 20 seconds.
   a. Prepare several conceptual designs-freehand.
   b. Choose the "best" solution and then prepare:
      1) Freehand sketches of the individual parts (detail sketches).
      2) An assembly drawing.
      3) A written description of the automobile.
      4) A letter of transmittal.

   Total time for project: 14 hours

2. Design an automatic feeder for an aquarium to meet the following requirements.
   a. Quantity of food per feeding should accommodate a normal fish population in a standard 20-gallon tank.
   b. Dry flake food will be used. Food should enter the aquarium at the same location for each feeding.
   c. A switch arrangement should be provided to turn on the aquarium lights at 6:00 a.m. and off at 6:00 p.m.
   d. The feeder must be fully automatic to operate seven days, two feedings daily (7:00 a.m. and 3:00 p.m.).

Prepare:

a. Freehand sketches of the components.
b. An assembly.
c. A written description of your design.
d. A letter of transmittal.

Total time for project: 18 hours.

3. Design a flour canister to meet the following general specifications:

   a. The canister is to stand on a counter or sink and dispense the premeasured quantity into a mixing bowl.

   b. The premeasured quantity should be variable from 1/8 cup to 1 cup in 1/8 cup increments.

   c. The canister should store five pounds of flour.

   d. The device must be moistureproof and able to be readily cleaned.

As an optional feature, it is proposed that an automatic sifter be incorporated into the design. The device may be either electrical or mechanical. The sifter should add as little as possible to the price of the canister.

Prepare:

a. Freehand details of the components.
b. An assembly.
c. A written description of your design.
d. A letter of transmittal.

Total time for project: 20 hours.

Several projects have been suggested by our "graduate-student" laboratory instructors who are aware of their own needs for equipment design related to their research activities. And, in fact, students propose their own projects. These are considered by the instructor and if found reasonable, with respect to degree of complexity and consumption of time, the student is encouraged to proceed. We have found that most students are thrilled with the opportunity to "be on their own" in expressing their creativeness through the solutions of conceptual design projects. There is no hesitancy, among the students, to criticize each others' work. There is a fine spirit of competition. You will be pleasantly surprised by the good results most of the students achieve.

Another dividend derives from the teaching experiences of the "graduate-student laboratory instructors. They have a true picture of a modern, science-oriented course in graphics and conceptual design. Moreover, as the "graduate-student" instructor moves into other areas of teaching - mechanics, design, etc., he knows how graphics and conceptual design are employed effectively in the solutions of problems that arise in those fields. And, additionally, he can use the course material in his own graduate studies as well as in his research activities.

I am quite convinced that through the activities of our graduate students both in teaching and research the stature of graphics and conceptual design is greatly enhanced.

I believe the "modus operandi" we are employing will contribute significantly to the development of fine engineering teachers and highly qualified engineers who are so desperately needed in this fast-moving technological era.

GRAPHIC TANTALIZER

Has anyone seen any practical application of a general warped surface in which the directrices are three curved lines?

GRAPHICAL ANALYSIS OF A NON-LINEAR DIFFERENTIAL EQUATION

Charles J. Baer — Associate Professor, Engineering Drawing, The University of Kansas
J. Russell May — Student, The University of Kansas

The equation having the form $x'' + v + \phi x = C$ represents a simple system with Coulomb damping. This describes the motion of a pendulum as well as the action of a synchronous motor. Heretofore it has been solved usually by Lienhard's graphical method, which is long and cumbersome.

At the suggestion of the chairman of the Department of Electrical Engineering (Dr. W.P. Smith), we approached the problem with the purpose of finding a quicker graphical solution. Although not all possible cases were covered in this project, we do believe that we have been able to devise a short, quick method that is just as accurate, if not more so, than Lienhard's Method.

Without going into the derivation of the equation at this point, we will simply say that the normalized solution to our equation $m \frac{d^2x}{dt^2} + k.v + \phi\,(x) = 0$ is $\frac{dv}{dx} = -\left(\frac{v + \phi\, x}{v}\right)$, or $\frac{dv}{dx} = -\left(\frac{v + \phi\, x}{v}\right) + A$ if there is a constant instead of zero on the right side of the equation. The complete derivation of this equation is shown at the end of this article.

With $C = 0$ and $\phi = \sin x$ our equation becomes $\frac{dv}{dx} = -\frac{H \sin x + v}{v}$ which can be considered to be a directional field. H is the ratio of the height (amplitude) of the sine curve to the length of a half cycle along the axis. Thus we were able to set up graphical solutions by varying the H values (Figures 3 through 6).

Using v for the vertical scale and $v + \phi x$ (actually $v + \sin x$) for the horizontal scale, we constructed the skeletal graphs, also passing a 45°line through the origin. The latter step was possible because we used a normalized equation, with values along the horizontal axis being equal to those on the vertical. Next a sine curve, with its amplitude plotted horizontally, was drawn utilizing the 45° line as its axis (Figure 1).

The direction of the field at a given point (Z_o, V_o) was found by drawing a line vertically (up or down) until it intercepted the 45° line at (Z_o, Z_o) as shown in Figure 1. Next a horizontal line was drawn until it intercepted the sinusoid at $Z_o + H \sin x, Z_o$. From this point a line parallel to the 45° line was drawn until it intercepted a horizontal line constructed through the original point (Z_o, V_o). From this point $(V_o + H \sin x, V_o)$ a line was drawn through the origin, and this line has the slope $\frac{V_o}{V_o + H \sin x}$. In order to get the correct slope at (Z_o, V_o) we drew a line perpendicular to the aforementioned line, as shown in Figure 1. Actually, by using two triangles, we were able to get the slope at any point without actually drawing the construction lines shown. The work sheet with small slope lines at selected points, sometimes called phase plane plots, looked somewhat as is shown in a small patch in the upper left hand part of Figure 1.

In order to draw a useful direction field a large number of these slope points must be plotted. Therefore we commenced a search for shortcut procedures with some success, as is shown in Figure 2.

Because of the fact that the slopes repeat themselves at quite a few points in the area we selected to portray graphically, we were able to draw the slopes at quite a few points almost simultaneously. For example, $\sin x = \sin 180° - x$.

Because we had divided the half cycle (180°) into ten intervals, we could quickly draw 45° lines corresponding to Z = 18°, 36°, 54°, 72°, and 90°. Then to find the slope at a given point we positioned a triangle to describe a line from the origin through the point whose horizontal grid line on graph paper, which passed through the given point, intercepted the 45° line corresponding to its Z value. (Figure 2) Next, we set a triangle perpendicular to the preceding line and drew the parallel slope lines at the several points by appropriate manipulation of the triangles. However, slopes were drawn only through points above the horizontal axis and with Z values from 0° to 360°. Later, when drawing continuous lines to describe the direction field, this pattern was merely shifted around to the other parts of the graph to supply the correct pattern.

After the necessary slopes had been drawn, we superimposed a sheet of tracing paper over the preceding work, and sketched in solid lines at fairly regular intervals. (Figure 3) Because we had slope lines at one-half inch intervals on graphs 15 inches square, the solid lines represent quite accurate representation of the slopes at all points on the graph. Figure 3 shows one of the four of the situations we solved. We obtained these continuous-line pictures for values of H equal to 0.1, 0.2, 0.4, 0.6, 0.8, 1.0, 1.25, 2.0, and 2.5, all with C = 0. This pretty much icluded the range of solutions we thought would be necessary for synchronous motor design.

As we suspect the case to be in many research situations, we were not always sure of what we were doing. For one thing, we did not know where to put the origin that was to be used for drawing slopes. Should we place it at the lower corner, on the graphical origin, at the upper corner, or where? In order to see how important this question was to our situation, we moved the origin up the 45° line to two alternate locations. Figure 4 shows the rather startling results obtained. Whereas the origin must be moved up along the 45° line the whole pattern shifted straight upward. Since moving the origin did not change the pattern but only shifted it up or down, we concluded that putting it at the graphical origin was as good a place as any to put it.

The next part of the project was to repeat the graphical procedures outlined above for cases having a constant in the equation. When the constant in the slope formula is A, then subtracting Av from the horizontal component of the line will give us

a slope: $\dfrac{V}{v + H \sin x - Av}$, and the slope

at the desired point: $\dfrac{-v + H \sin x - Av}{v}$

or $-\dfrac{v + H \sin x}{V} + A$. The term Av can be

treated in this manner by shifting the origin a distance of Av to the right along the horizontal axis, as shown in Figure 5.

One interesting case which developed is that where H = 1 and A = 1, shown in Figure 6. Here the lines do not diverge or converge, and that the system is periodically stable--unlike any of the other situations that were analyzed.

Several additional graphical techniques were used in this project, but only the main ones have been shown in order to keep this article to a minimum length. We invite comments from any reader on this or any related subject.

equation

$$m \frac{d^2X}{dt^2} + \phi(v) + K(x)=0$$

$$\frac{d^2x}{dt^2} + \frac{1}{m}\phi/(v) + \frac{K}{m}(x)=0$$

$$t = a\tau$$

$$dt^2 = a^2 d\tau^2$$

$$\frac{d^2x}{a^2 d\tau^2} + \frac{1}{m}\phi\left(\frac{dx}{d\tau}\right) + \frac{K}{m}(x) = 0$$

$$a = (m/k)^{1/2}$$

$$\frac{d^2x}{d\tau^2} = \frac{dv}{d\tau} = \frac{dv}{dx} \cdot \frac{dx}{d\tau} = v\frac{dv}{dx}$$

$$v\frac{dv}{dx} + \frac{a}{m}\phi(v) + x = 0$$

for a = 1,

$$v\frac{dv}{dx} = -\phi(v) - x$$

$$\frac{dv}{dx} = -\frac{\phi(v) + x}{v}$$

using a similar derivation, one can find the normalized solution to

$$m\frac{d^2x}{dt^2} + K.v + \phi(x) = 0$$

which is

$$\frac{dv}{dx} = -\frac{v + \phi(x)}{v}$$

When there is a constant C in the equation instead of a zero, the solution is

$$\frac{dv}{dx} = -\frac{v + \phi(x)}{v} + A$$

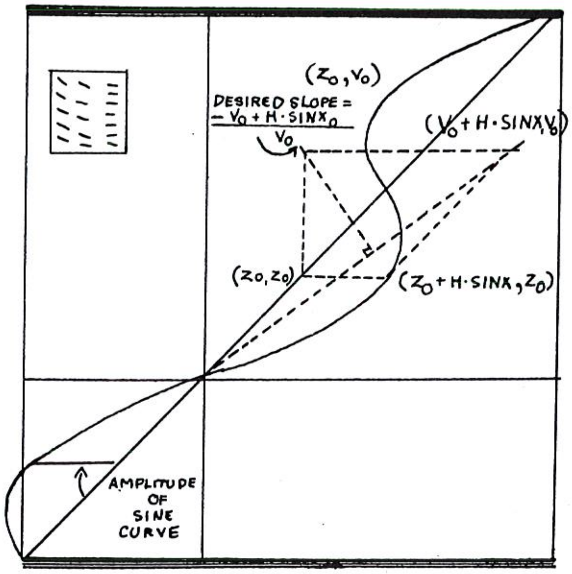


Fig. 1 Sketch of method used to obtain slopes.

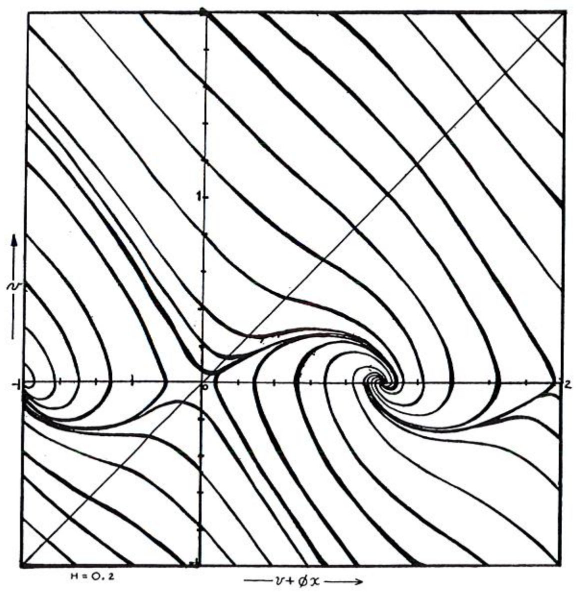


Fig. 3 Finished continuous-line patterns of the equation for values of H = 0.2

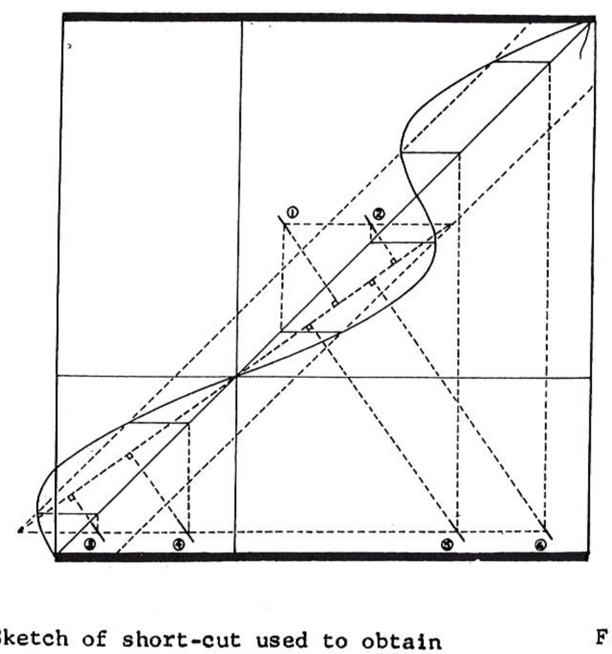


Fig. 2 Sketch of short-cut used to obtain slopes of several points simultaneously.

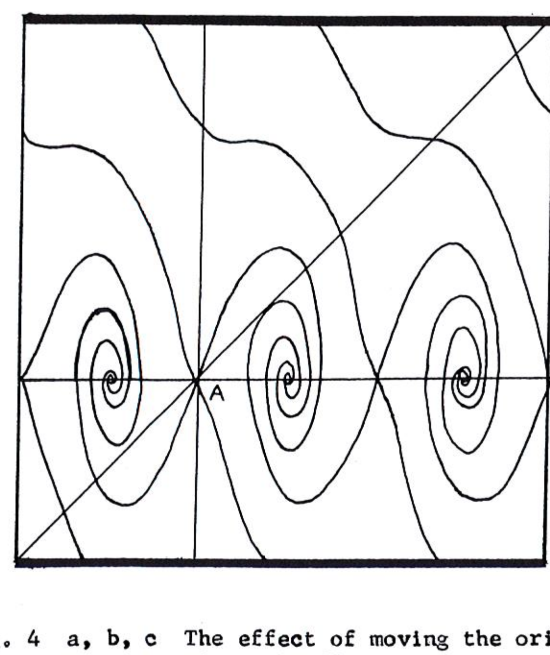
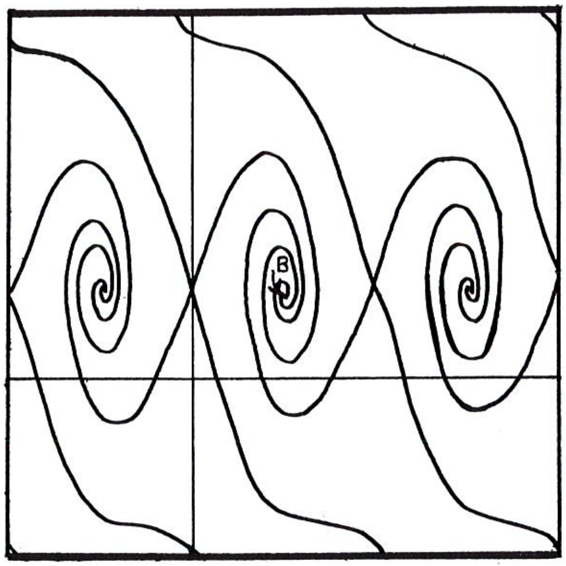
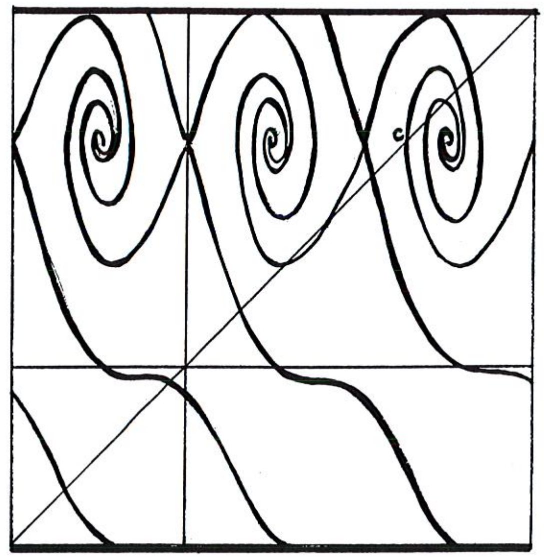


Fig. 4 a, b, c The effect of moving the origin along the 45-degree line



4b



4c

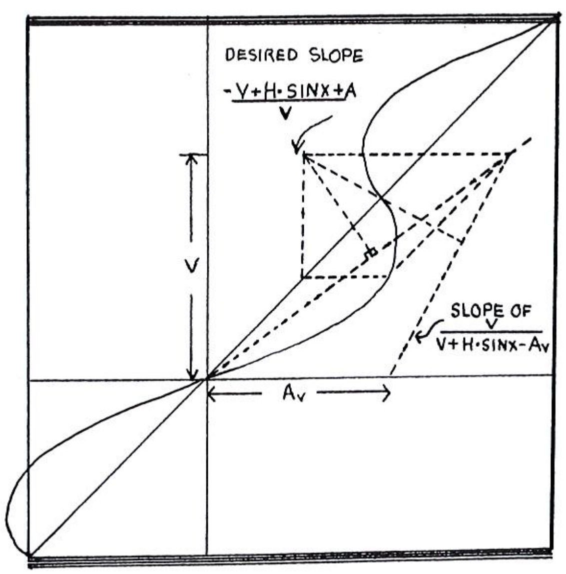


Fig. 5 Sketch of method used to add constant, A, to graphical solution.

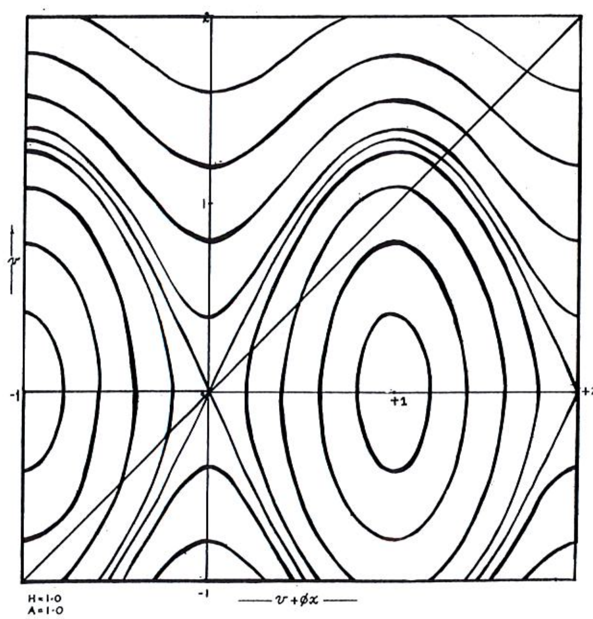


Fig. 6 Finished continuous-line pattern of the case H = 1.0 and A = 1.0.

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PROJECTIONS FOR A GEODESIC SPHERE

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History shows the use of rectangular shapes in structure more frequently than curved or spherical shapes because the shape of the sphere is difficult to construct and difficult to combine with other architectural elements. Straight lines are easier to draw and flat planes are easier to construct than those which are curved. Until the discovery of calculus, determining a sphere or a part of a sphere was at best a linear approximation. The sphere is one of the geometrical shapes included in nature's growth and function patterns. Concentric circles, concentric circles, logarithmic spirals, and the series show nature's strong preference for order and geometry.

Technical innovations and military needs have accelerated the use of this shape in the last decade. Future uses of space frames and of large-volume enclosures will require new structural devices and systems of construction. The further development of light-weight structural materials with ultra-high strengths will make these new systems possible. In such an era perhaps the dome will emerge as an answer to new demands and new needs. Before the end of this century, communities may be completely enclosed with dome-shaped space frames, artificially climated and inter-connected with other space frames that enclose entire geographical areas. The potentials of such structures and their uses are limited only by man's imagination and his ability to continue to discover the technical means.

SPHERE

The dome, or, more exactly, the sphere, is a double curved surface and theoretically cannot be developed. It can be approximated and several means of projection have been devised for this purpose. General surface distortions are evident in these projections unless the sphere is broken into many small areas. Two methods of approximation that are used in descriptive geometry to divide the sphere into cylindrical or conical segments are called the gore or zone methods. The accuracy of the development increases as the number of zones or gores is increased, and the greater the need for a near-perfect sphere, the greater the number of separate areas that must be developed.

A structural system based on this appoximation requires curved members that serve as parallels and meridians. Usually the meridians become the major structural elements and the parallels are reduced to straight-line bridging between the meridians. If the meridians become short, straight-line framing members, their joints must be designed to transfer the loads as well as to carry the intermediate bridging which is necessary for lateral support. The rectangular voids formed by the intersection of the numbers are not self-rigid and depend entirely upon rigid connections for stability. The addition of a covering skin to the structure will increase this stability although there are few covering materials that can satisfy all needs in one package. If diagonal tension members are added to the basic frame to triangulate the system, then perhaps the appropriateness of the structure should be questioned.

If it is sufficient for straight lines to approximate a sphere, then the chords of geodetic arcs are satisfactory. These chords can be transferred to straight-line structural elements and the entire system reduced to the basic self-rigid component of the triangle in plan, or the tetrahedron in space.

The sphere covers the greatest volume with the least surface area and the triangle is the basic structural component. These two can be combined graphically to produce a sphere or a portion of a sphere in which the basic component is a triangle. If the great circles of a sphere are reduced to a series of arcs and these arcs connected by their chords, a sphere will result made of chords of the lines of geodesy, more popularly referred to as a geodesic sphere. The easiest graphical method of constructing a geodesic dome uses orthographic projection, descriptive geometry and a known polyhedron to project to a circumscribed sphere or to an inscribed sphere, either projection resulting in a dome composed of chords of great circle arcs.

POLYHEDRA

The number of regular polyhedra which can be inscribed within a sphere and have their vertices equidistant apart on the surface of the sphere, or have a sphere inscribed within them so that the sphere is tangent at the center of the faces, with these points of tangency equidistant apart, is limited to five. These five polyhedra, with equal solid angles, equal regular faces and equal length of sides or angles are known as the Platonic bodies. They are the tetrahedron, hexahedron, octahedron, dodecahedron and icosahedron. Each polyhedra is related by Euler's formula which states that the number of sides plus two equals the number of vertices plus the number of faces. (S plus 2 = V plus F). These five solids are related to each other in several interesting ways. For instance, there is a reciprocal relationship between the octahedron and the cube and between the dodecahedron and the icosahedron. If lines are drawn between the center of the faces of one of these solids, its reciprocal is produced. If lines are drawn between the center of the faces of the tetrahedron it will reproduce itself. Arranging the vertices of one body to fall at the center of another body's face will establish a nesting relationship. By connecting the center of the faces of the icosahedron a dodecahedron is produced. Connecting across the faces of the pentagons of the dodecahedron establishes a cube. Drawing both diagonals of the faces of the cube produce two interlooking tetrahedra and connecting the points of intersection of the diagonals of the cube produce an octahedron. Other similar relationships can be established including the ϕ series which uses the "golden rectangle."

There are also thirteen semi-regular polyhedra, known as the Archimedian solids that can be inscribed within a sphere, with tangent points at their vertices, or can have a sphere inscribed within them. They have equal length sides and regular polygonal faces, although all the faces are not identical. They are the truncated tetrahedron, truncated octahedron, truncated cube, truncated dodecahedron, truncated icosahedron, rhombicuboctahedron, cuboctahedron, rhombicosidodecaedron, icosidodecahedron, truncated cuboctahedron, snub cube, snub dodecahedron, and the truncated icosidodecahedron. For clarity the hidden lines have been omitted.

Some prisms and anti-prisms, can be placed within a sphere and have their vertices tangent to the sphere and equidistant.

DRAWING THE FIVE SOLIDS

The construction of principle views of the five platonic solids are illustrated in 2, 3, 4. The three orthographic views of each solid have been designated vertex view, face view and edge view. In each case the observer is looking normal to one of the vertices, one of the faces or one of the edges of the solid.

Three views of the tetrahedron, hexahedron and octahedron with their circumscribed and inscribed spheres are shown in figure 2. Since the construction of these views is not difficult it has been omitted.

The diagonal of a face of the hexahedron is equal in length to the edge of the tetrahedron. This diagonal is true length in both the face view and vertex view of the hexahedron. The vertex view of the hexahedron is shown as an isometric projection so that the diagonal of its upper face is true length and equal to the diagonal in the face view.

Three views of the dodecahedron, their related construction views, and the circumscribed and inscribed sphere are shown in figure 3. All vertices of the dodecahedron touch the sphere, although this is visible only in the edge view. In the other two views the points of tangency between the sphere and the vertices are located in space either above or below the plane that contains the true circumberence of the sphere.

The thirteen Archimedian bodies shown in figure 1 can be obtained by similar construction views, although it is easier to draw the Platonic body first and then connect mid points or third points on the edges or faces of the Platonic bodies. The name of the body indicates the method of obtaining its shape. There are reciprocal relationships between these bodies and one may pass from one body to another either directly or in a series of related steps. As an example: starting with a tetrahedron and connecting the midpoints of the edges will result in an octahedron; connecting the mid points of an octahedron will produce a cuboctahedron; connecting the third points of an octahedron will produce a truncated octahedron; connecting mid points of a cubotahedron will produce a rhombicubotahedron. Another example: starting with an icosahedron and connecting the third points of the edges will produce a truncated icosahedron; connecting mid points of an icosahedron will produce an icosidodecahedron; connecting mid points of the icosidodecahedron will result in a rhombicosidodecahedron. Similar relations are found by connecting points on the faces or edges of the various bodies.

DRAWING THE DOME

STEP 1.

A polyhedron is selected. Any of the platonic and Archimedian shapes on figure 1 can be graphically projected into a dome that follows the lines of geodesy. There are two reasons for using the icosahedron in these drawings. First, its shape is visually more spherical than that of the tetrahedron, hexahedron or octahedron and second, its equilateral triangular faces are easier to draw than the pentagonal faces of the dodecahedron.

STEP 2.

Figure 5 shows the orthographic relationship between the vertex view, the edge view and the face view of an icosahedron.

Figure 6 shows the orthographic relation between the vertex view and edge view of the icosahedron and its circumscribed sphere.

STEP 3.

After constructing and orthographically arranging these views, all the vertices are numbered. It is recommended that some numbering system can be employed as an aid in projection

STEP 4.

The mid points of the edges are then connected to produce the icosidodecahedron on figure 7.

There are two conditions which will appear throughout this method of projection. Connecting points will result in a reduced-size body and the loss of the original sphere. After connnecting points the body is truncated and the surfaces become flat since the vertices of the originating body are removed. The edges of these surfaces can become chords of great circle arcs although the body does not have a particular spherical form, and these surfaces have introduced one or more plain geometric shapes. Further, these geometric shapes are usually not self-rigid and are better reduced to triangular components. In the icosidodecahedron of figure 7 there are triangles and pentagons. The pentagon can be triangulated as a flat surface or, since it is better to have all lines become chords of great circle arcs, the intersection of the triangles in the pentagon is projected to the surface of a sphere. This will place all five corners of the pentagon and the common intersection of the five triangles on the surface of a sphere. This projected intersection becomes a vertex of the pentagon.

STEP 5.

Fig. 8 shows the first projection (icosidodecahedron) with a new circumscribed sphere and the projected vertices. The sphere is drawn so that it is tangent to the extreme points in the edge view (the same view in which the original sphere appeared tangent). Vertices are then projected perpendicular from the pentagons until they are tangent to the sphere, locating the new vertices on the surface of the sphere and making the edges chords of geodesic areas. The corners of the geometric plane (in this case the pentagons) are tangent to the sphere and the perpendicular erected to position the vertex is always drawn to the center of the sphere. This is true whether the geometric plane appears normal or inclined. In the plan view of figure 8 (to eliminate a duality of terms in the explanation, the vertex view will be refered to as plan and the edge view referred to as elevation) the center pentagon (1, 2, 3, 4, 5) appears true size and shape in plan and as an edge in the adjacent view.

STEP 6.

The center (6) of the pentagon is located in the plan view and the five triangles are drawn.

STEP 7.

The center of the pentagon is located in the elevation view on the surface of the pentagon and a line is drawn through this point and the center of the sphere until it intersects the sphere. This locates the vertex (6) on the surface of the sphere. The triangles are then drawn in the elevation connecting point 6 with points 1, 2, 3, 4, 5.

STEP 8.

The vertices of the remaining eleven pentagons must be located in both views. To simplify the drawings all hidden lines on figure 8, and the sheets that follow are omitted. Omitting the hidden pentagons leaves six pentagons visible in the plan view and eight pentagons visible in the elevation view. After locating the vertex (6) in the elevation view the vertex for the same pentagon on the opposite hemisphere is located. A line is drawn in the elevation view from point 6 through the center of the sphere and extended until it intersects the sphere (6B). The vertex of the bottom pentagon is at this intersection. Note from figure 6 that this bottom pentagon is rotated 180^{o} from the top pentagon. In locating additional vertices it is useful to remember that for every point on one hemisphere there is a related point on the other hemisphere, and a straight line through these points will pass through the center of the sphere.

STEP 9.

The next vertex to locate is for pentagon 1, 7, 8, 9, 10. This pentagon also appears as an edge in the elevation view and its center can be located by measurement in the edge view from pentagon 1, 2, 3, 4, 5.

STEP 10.

A line is then drawn in the elevation view through the center of pentagon 1, 7, 8, 9, 10 and the center of the sphere until it intersects the sphere in both hemispheres (11 and 11B). This locates the vertex for these pentagons.

STEP 11.

To locate the vertex (11) in the plan view, the line 6-1 is extended into pentagon 1, 7, 8, 9, 10; this is the perpendicular bisector of line 8-9. A second line is orthographically projected from point 11 in the elevation. The intersection of these two lines in pentagon 1, 7, 8, 9, 10 locates the vertex (11) in the plan.

STEP 12.

The remaining four pentagons in the plan are all at the same elevation on the sphere as pentagon 1, 7, 8, 9, 10. To locate their vertices a horizontal cutting plane is passed through the sphere and through vertex 11.

STEP 13.

This cutting plane is located in the plan view and the circle of its out is drawn.

STEP 14.

Lines are then extended from vertex 6 into the four remaining pentagons. The vertices are located at the intersection of these extensions (perpendicular bisectors) and the cutting plane's circle.

STEP 15.

These vertices are orthographically projected to the elevation view cutting plane, and then extended (through the center of the sphere) to the lower hemisphere.

STEP 16.

All the visible vertices are located, the triangles are drawn and the first projection geodesic sphere is complete.

On figure 8 the plain geometric shapes are shown as black lines and the triangles that meet at the vertices are shown as gray lines. If projections are made to determine true lengths, it will be found that all the black lines are one length and all the gray lines are another length. The triangles composing the vertices are isosceles. By neglecting the width of the inked lines careful observation will reveal a series of hexagons. Subsequent projections will show that as a result of combining the triangles into larger groupings, the hexagon becomes the basic shape used in the geodesic dome. Since the triangles composing these hexagons are not equilateral triangles, then the hexagons are not regular hexagons and the sum of the angle at the vertex of the triangles is less than 360^{o}. In any geodesic sphere the arrangement of the hexagons will always leave 12 voids which can only be closed with pentagons. Euler's formula also applies to this polyhedron.

Although the vertex view and the edge view of Fig. 8 have been labeled plan and elevation, these two could be interchanged, and either view can be developed as a plan or an elevation. A ground line can be located at any height in the elevation. However, the most logical location would follow the structural members and not disrupt the triangular continuity. If the elevation is cut as a hemisphere, the ground line is level. If it is placed at any other elevation the dome will not produce a level line but will rest on several points.

Figure 9 is the second projection of the original icosahedron. It is obtained by connecting the mid points of the edges of the first projection. The shape produced is similar to the one of the first projection, figure 7, except there is an increase in the number of elements and the number of geometric planes. The geometric planes are triangulated and the common intersection of the triangles are projected to vertices on the surface of the sphere.

All the projections in these drawings have been outward to the circumscribed sphere. Similar results can be obtained by projecting inward to the inscribed sphere. Graphically, the outward projection is easier since an inward projection will introduce hidden lines from the geometric planes to their vertices. The method of circumscribing the sphere and of locating the vertices for this second projection is identical with the method described for the first projection. The result is shown on figure 10. The geometric planes, resulting from the connection of mid points, are shown as black lines and the vertices are shown as gray lines. If true lengths or true size and shape are found it will be noticed that the identical shapes of figure 9 will produce identical sizes, shapes and lengths. The number of different lengths of lines necessary to compose this sphere has increased over the previous projection. However, two sides of each of the triangles that compose the hexagon, pentagon and triangle of figure 9 will determine all the lengths required. A series of related hexagons and the necessary pentagons form the sphere. Six pentagons are visible in plan and eight pentagons are visible in elevation. A ground line may be drawn in the elevation at any of several locations.

The illustrations are all mid-point connections, although any other equal division of third points, quarter points, etc., would produce similar results. The lines of the icosahedron are reduced to smaller elements more rapidly if additional equal divisions are used.

Continuing on figure 11 is the third projection, obtained by connecting the midpoints of the edges of the second projection sphere of figure 10.

Circumscribing the sphere and locating the vertices gives the hemisphere shown on figure 12. The planes are shown as black lines and the triangulations are shown as gray lines. A ground line can be placed to obtain a dome of various span-rise ratios.

Figure 13 is a section of the hemisphere of figure 12. This section was taken immediately above the pentagons. The elements are emphasized to show arrangement of hexagons in black with their vertices in gray. The dome touches the ground line at five points and the pentagonal shape of the originating icosahedron is quite apparent.

Another possible pattern is shown on figure 14 where the diamonds of the hexagons have been emphasized. A cross member would be necessary to complete the triangles.

This method of connecting equally spaced points to obtain smaller units and shorter lengths may be utilized until the desired sizes are obtained, and additional patterns may be found and emphasized. The designer may quickly determine, with freehand overlays, to what extent the original icosahedron should be divided. For study and for models it is not necessary to develop more than the different geometric shapes. This is accomplished by limiting the graphics to one-twentieth of the icosahedron, that is, a development of one of the equilateral triangles. The complete development of one equilateral triangle will give all the lengths, sizes and angles necessary to determine a sphere.

Another method of developing a dome is by projecting equilateral triangles outward until they coincide with the surface of the circumscribed sphere as illustrated on figure 15. The icosahedral triangles become equilateral spherical triangles which in turn are reduced to straight-line members that are chords of great circle arcs.

STEP 1.

The plan contains a vertex view of the originationg icosahedron, the third view is a construction for determining true lengths.

STEP 2.

In constructing the true-length view, the equilateral triangle is drawn first, equal in size to the equilateral triangles which form the icosahedron.

STEP 3.

A circle is drawn around the triangle tangent to its corners (1, 2, 3). This circle represents that portion of the sphere which is cut by a plane that passes through the sphere parallel and tangent to the face of one of the equilateral triangles of the icosahedron.

STEP 4.

Perpendicular bisectors are drawn through lines 1-2, 2-3 and 3-1.

STEP 5.

With a radius equal to the radius of the circumscribed sphere and from a point on each perpendicular bisector, the three arcs 1-2, 2-3 and 3-1 are drawn through the corners of the triangles. (These arcs may be drawn either inside or outside of the triangle. However, for clarity it is best to keep them outside of the triangle.) These are arcs of great circles and represent the curve the edges of the triangle must approximate to become chords of the great circles. The view is normal to each arc after it has been rotated about an axis which coincides with the straight line 1-2, 2-3 and 3-1. Each edge of the triangle is being projected into its curved form separately, although one construction view is being used for all three edges.

STEP 6.

The three arcs are divided into equal parts.

STEP 7.

These divisions are projected toward the center of the radius of the arc, which represents the center of the sphere, until they intersect the straight lines of the original triangle, locating points 4, 5, 6, 7, 10, 11, 13, 14 and 15.

STEP 8.

These points are connected dividing the equilateral triangle into 16 triangles. The small triangle formed at the intersection of these connecting lines is not the result of drafting error but the overlapping of chord lengths of spherical triangles that have been forced to lay in flat plane.

If the length of projection of a point is measured from the arc to the straight line, it will give the distance that point must be elevated to become tangent with the sphere. The distance at the corners 1, 2 and 3 of the triangle will be zero and this is correct since, by definition, these points must already be tangent with the sphere. If the distance between points 2-4, 4-5, 5-6 and 6-3 is measured along the arc, the true length of the chord is found. Line 1-2 equals line 2-3 equals line 3-1 and arc 1-2 equals arc 2-3 equals arc 3-1. Thus the elevation and location of any point, or the chordal distance between any two points, is the same for the corresponding point or points on the other two arcs or lines. The lengths and changes in elevation of the periphery of the triangle is now known. A similar process determines the lengths and heights of the internal members.

STEP 9.

The line through points 7-10 is extended until it intersects the circle.

STEP 10.

An arc is drawn through these intersection points with a radius equal to the radius of the sphere. This arc is to line 7-10 and the points 7, 8, 9, 10 the same as arc 2-3 is to line 2-3 and points 2, 4, 5, 6, 3.

STEP 11.

To determine the elevation of point 8, and its location on the arc, project the point on a line that passes through point 8, through the center of the radius of the arc, and intersects the arc.

STEP 12.

Project points 7, 9 and 10 in the same

manner. This locates all the points of line 7-10 on the sphere. The elevation of the points may now be measured. As a check, the distance from point 7 to its arc should be exactly the same as the distance from point 7 to arc 1-2. This measurement should be the same for points 4, 6, 7, 10, 14 and 15. The change in elevation and the distance between points is now known for 21 of the 30 lines which compose this spherical triangle.

STEP 13.

The line through points 11-13 and the line through points 14-15 is extended to intersect the circle, their arcs are drawn, the points projected, and the construction drawing is complete.

Actually not all of the arcs and projections were necessary. The first two arcs would give all the needed lengths and changes of elevation since the seven triangles along line 2-3 are repeated along lines 1-2 and 3-1.

STEP 14.

All of the points may be located in plan and elevation, using cutting planes and orthographic projection and remembering that in projecting any point from the plane triangle to the sphere, the line of projection will always pass through the center of the sphere regardless of the position or view of the triangle.

Edge 1-2 is true length in elevation and points 7, 11 and 14 may be located on this line in the elevation view and projected to the sphere.

STEP 15.

Horizontal cutting planes are passed through the projected points 7, 11 and 14 in the elevation view.

STEP 16.

The circles of the cutting planes are drawn in the plan view.

STEP 17.

Points 7, 11 and 14 are located in plan for all five triangles.

STEP 18.

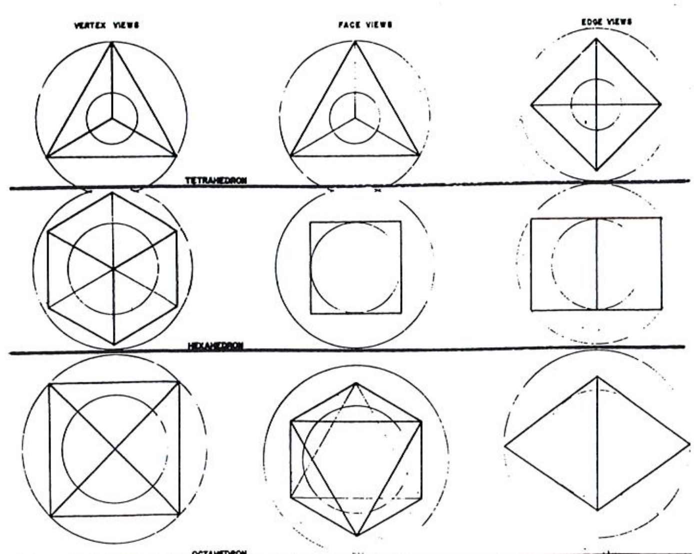
These points are projected back to the cutting planes in the elevation view.

STEP 19.

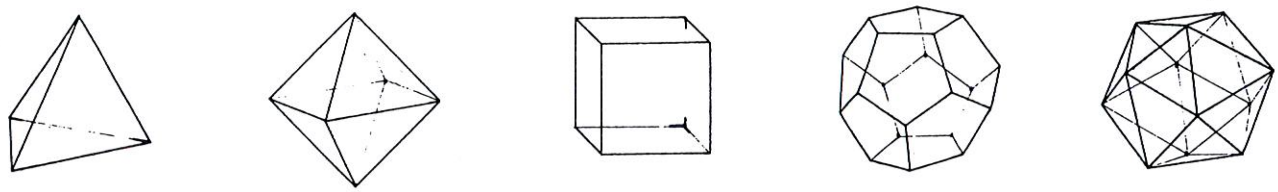
The balance of the points are located in a similar manner and the drawing is completed. All areas are triangulated and all points fall on the surface of the sphere.

To what extent these shapes are developed and for what purposes they are used is left to the discretion of the designer. This description is to establish a method for graphical communication.

figures 1—15



TETRAHEDRON HEXAHEDRON OCTAHEDRON 2



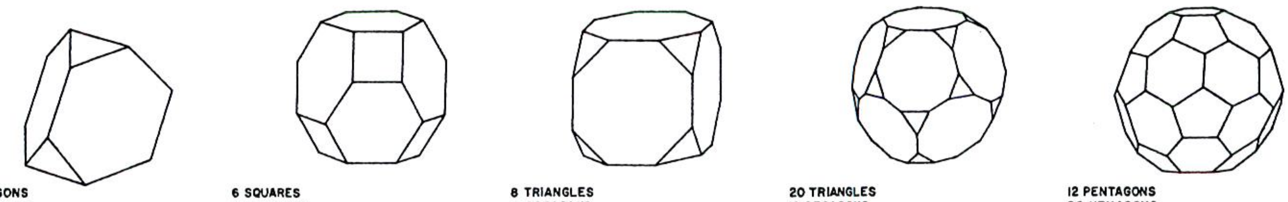
4 TRIANGLES
TETRAHEDRON

8 TRIANGLES
OCTAHEDRON

6 SQUARES
HEXAHEDRON

12 PENTAGONS
DODECAHEDRON

20 TRIANGLES
ICOSAHEDRON



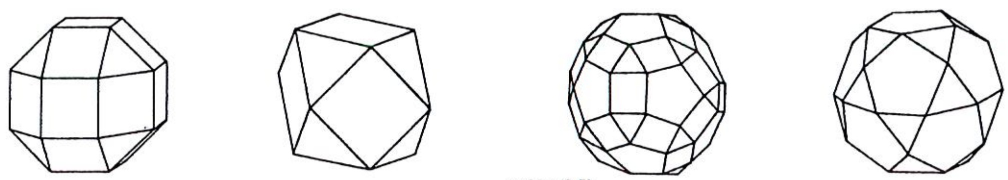
4 HEXAGONS
4 TRIANGLES
TRUNCATED TETRAHEDRON

6 SQUARES
8 HEXAGONS
TRUNCATED OCTAHEDRON

8 TRIANGLES
6 OCTAGONS
TRUNCATED CUBE

20 TRIANGLES
12 DECAGONS
TRUNCATED DODECAHEDRON

12 PENTAGONS
20 HEXAGONS
TRUNCATED ICOSAHEDRON

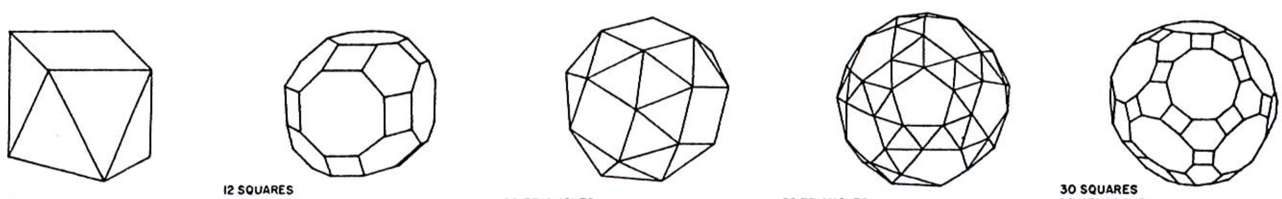


8 TRIANGLES
18 SQUARES
RHOMBICUBOCTAHEDRON

8 TRIANGLES
6 SQUARES
CUBOCTAHEDRON

20 TRIANG_ES
30 SQUARES
12 PENTAGONS
RHOMBICOSIDODECAHEDRON

20 TRIANGLES
12 PENTAGONS
ICOSIDODECAHEDRON



8 TRIANGLES
2 SQUARES
ANTI-PRISM

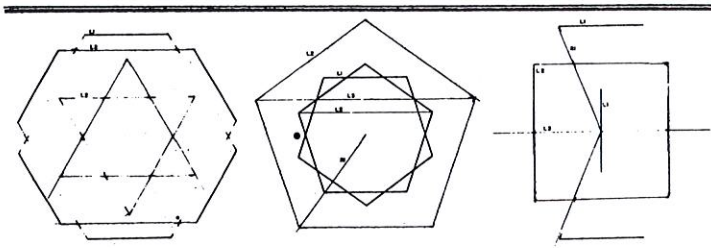
12 SQUARES
8 HEXAGONS
6 OCTAGONS
TRUNCATED CUBOCTAHEDRON

32 TRIANGLES
6 SQUARES
SNUB CUBE

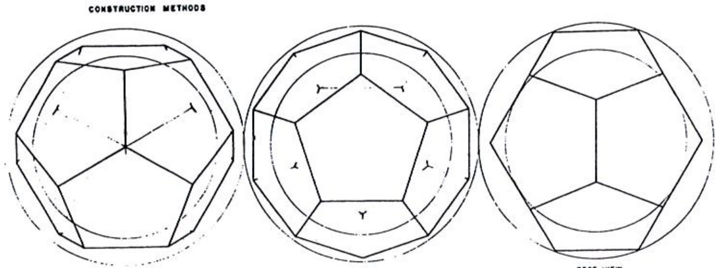
80 TRIANGLES
12 PENTAGONS
SNUB DODECAHEDRON

30 SQUARES
20 HEXAGONS
12 DECAGONS
TRUNCATED ICOSIDODECAHEDRON

SOLIDS 1.

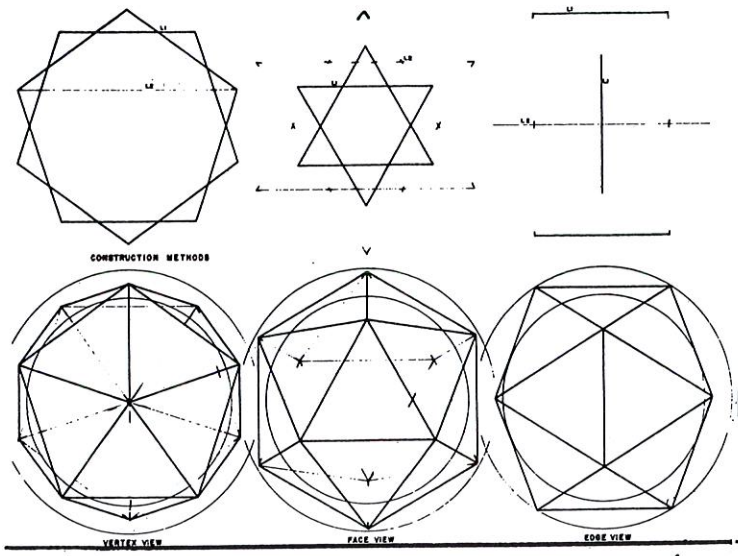


CONSTRUCTION METHODS

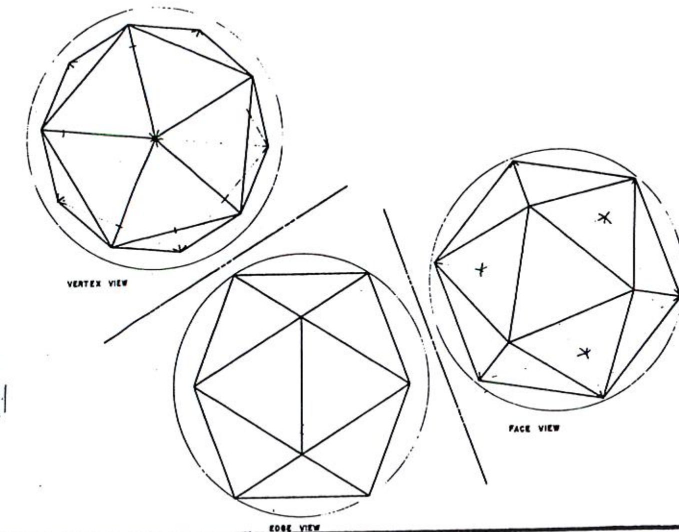


VERTEX VIEW FACE VIEW EDGE VIEW

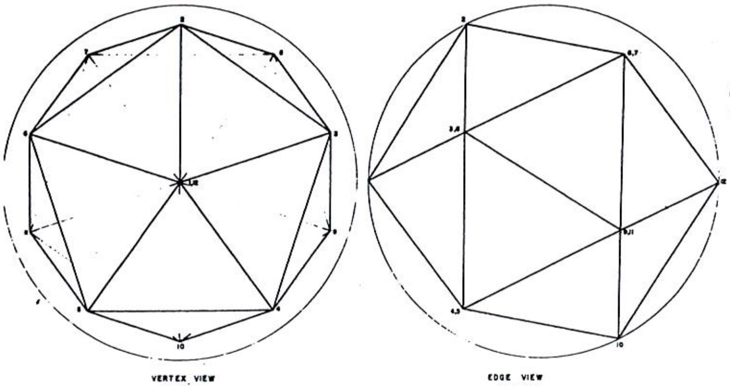
DODECAHEDRON 3



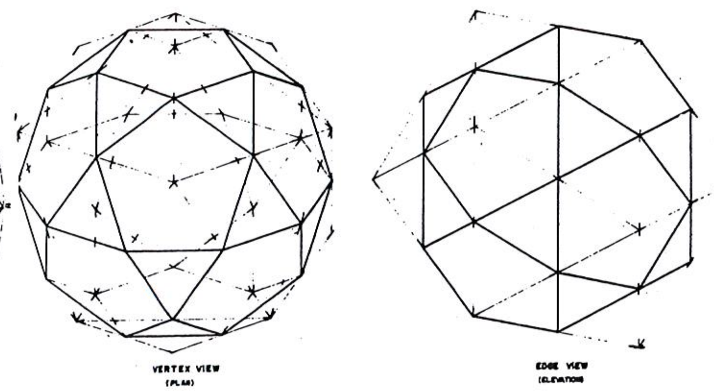
ICOSAHEDRON 4



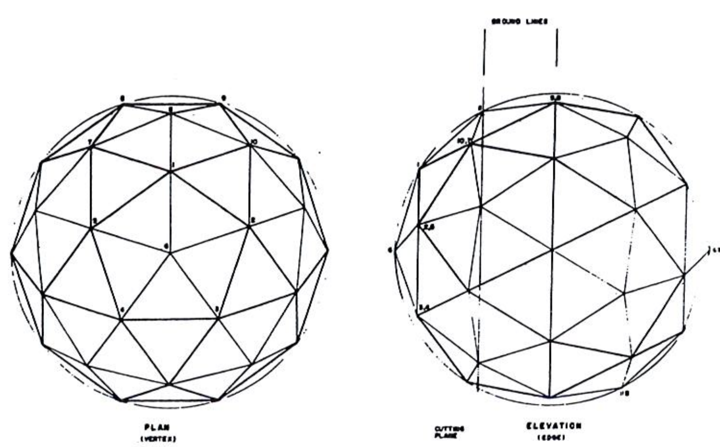
ORTHOGRAPHIC PROJECTION 5



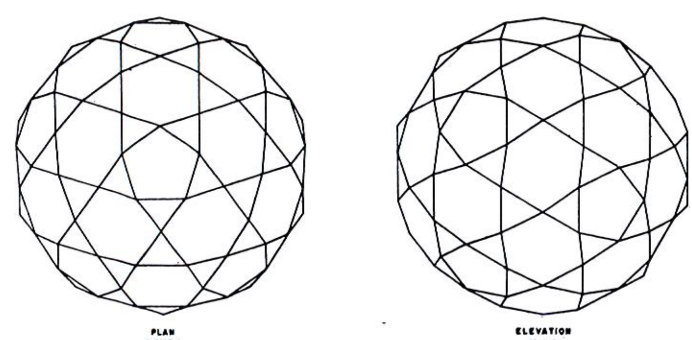
ICOSAHEDRON 6



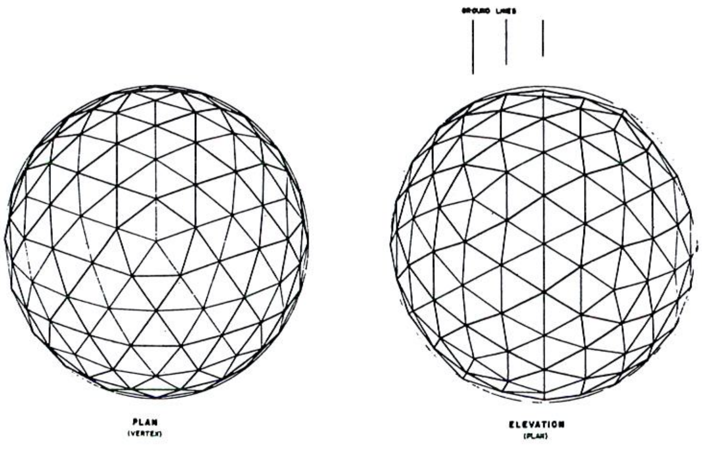
FIRST PROJECTION 7



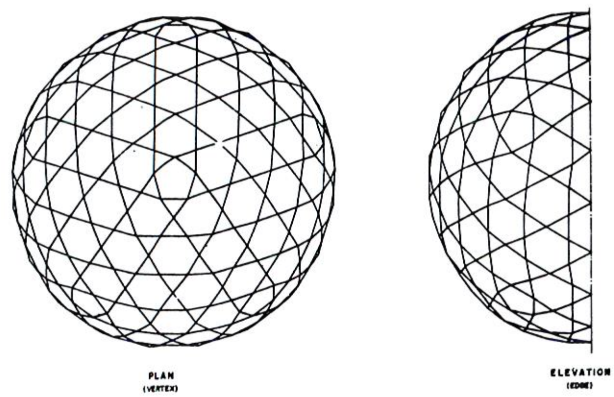
FIRST PROJECTION SPHERE 8



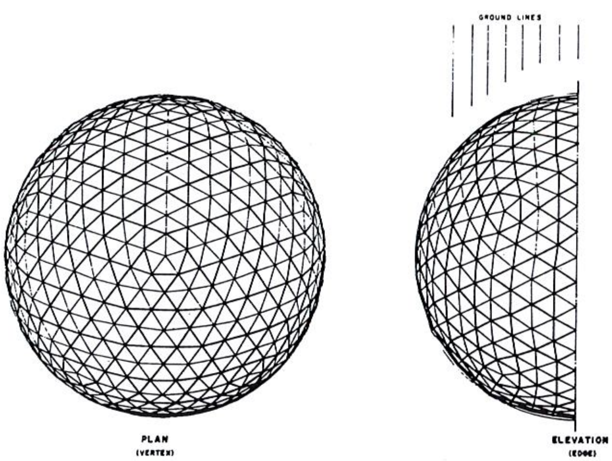
SECOND PROJECTION 9



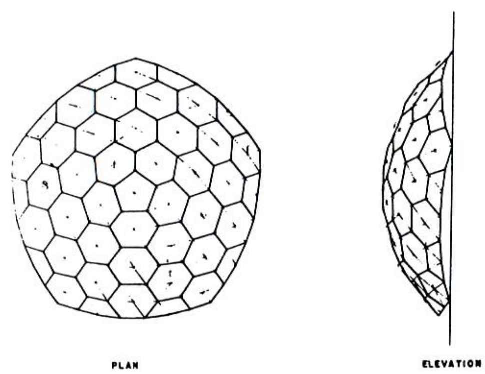
SECOND PROJECTION SPHERE 10



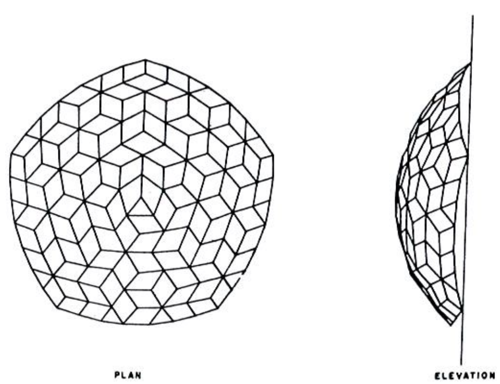
THIRD PROJECTION 11



THIRD PROJECTION SPHERE 12



PATTERN 13



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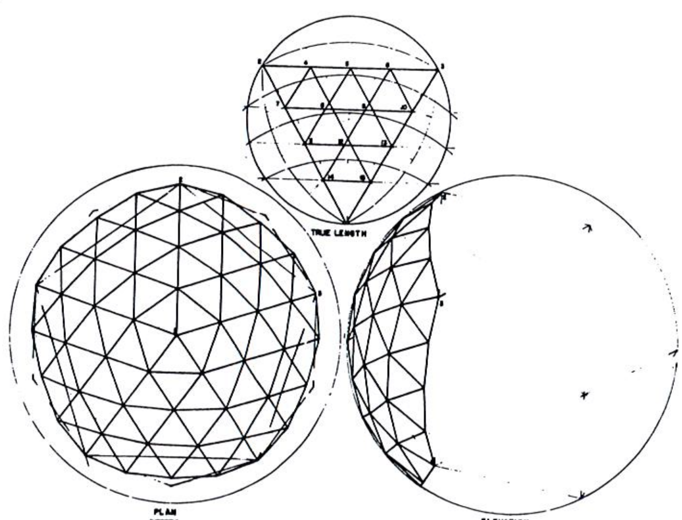

15

THE GRAPHIC SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS WITH MORE THAN TWO VARIABLES

by M. E. Arthur

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Catalog Descriptors:

1. Simultaneous equations
2. Equations
3. Graphical methods
4. Complex variables
5. Real variables

ABSTRACT

This paper presents a graphic method for solving simultaneous equations of more than two variables. Several three-variable solutions and a four variable solution are demonstrated. Method extension beyond four variables is defined.

The solution of two simultaneous linear equations by plotting the equations on one coordinate system is taught in public high schools. For example, equations 1a and 1b are plotted in figure 1. Note that $x = 9$ and $y = 2$ satisfy both of these equations:

$$x + y = 11 \tag{1a}$$

$$x - y = 7 \tag{1b}$$

In figure 2 the same two equations are plotted and a third equation has been added:

$$2x + y = 6 \tag{1c}$$

In figure 2 we see that equations 1a and 1c have the common solution where $x = -5$ and $y = 16$ and that equations 1b and 1c have the solution $x = 4\frac{1}{3}$ and $y = -2\frac{2}{3}$.

Equations 1a, 1b, and 1c are not the only equations which might be graphed as in figure 2. Consider equations 2a, 2b, and 2c when $z = 0$:

$$x + y + c_1 z = 11 \tag{2a}$$

$$x - y + c_2 z = 7 \tag{2b}$$

$$2x + y + c_3 z = 6 \tag{2c}$$

In equation 2a, 2b, and 2c, note that so long as $z = 0$, the coefficients of z, the subscripted c's, have no effect.

Before these equations can be plotted with z equal to any value other than zero, numerical values must be substituted for the c's. Equations 3a, 3b, and 3c are the same as equations 2a, 2b, and 2c with $c_1 = +2$, $c_2 = +4$, and $c_3 = -2$:

$$x + y + 2z = 11 \tag{3a}$$

$$x - y + 4z = 7 \tag{3b}$$

$$2x + y - 2z = 6 \tag{3c}$$

Equations 3a, 3b, and 3c are plotted in figure 3 for $z = 0$ and $z = 1$. In figure 2 the three lines enclose a triangle whose vertices are (9, 2), (-5, 16) and $(4\frac{1}{3}, -2\frac{2}{3})$. Similarly, in figure 3 the lines for $z = 0$ enclose the same triangle and the lines for $z = 1$ enclose the triangle (6, 3), (-1, 10) and $(3\frac{2}{3}, \frac{2}{3})$. One might find the solution of equations 3a, 3b, and 3c graphically by substituting other values of z in search of the one value that would make the three lines for these equations pass through some one point — that is make the triangle reduce to zero. However, there is a better way.

Consider the three possible pairs of equations from equations 3a, 3b, and 3c. For equations 3a and 3b, $z = 0$ at (9, 2) and $z = 1$ at (6, 3). An extended line, Z_{ab} drawn through these two points as in figure 4, gives us all the points for which equations 3a and 3b will be satisfied for various values of z. Equation 3a multiplied by 2, $2x + 2y + 4z = 22$, minus equation 3b gives $x + 3y = 15$. $x + 3y = 15$ is the equation of Z_{ab}.

Similarly for equations 3a and 3c, $z = 0$ at (-5, 16) and $z = 1$ at (-1, 10). A line, Z_{ac}, through these points will give those points for which equations 3a and 3c will be satisfied for various values of z. Equation 3a plus equation 3c gives $3x + 2y = 17$ which is the equation of Z_{ac}. Similar statements can be made for equations 3b and 3c producing Z_{bc}. The symbol Z_{ij} will be used hereafter to mean some or all such lines as Z_{ab}, Z_{ac}, etc., which result from the construction discussed above. The Z_{ij} lines are drawn in figure 4 superimposed on a duplication of figure 3. All three intersect at $x = 3$, $y = 4$. Consider the three pairs of similar triangles with their apexes at (3, 4) and their bases on the pairs of lines plotted for equations 3a, 3b, and 3c. Note that z changes the same amount, positive in this case but not always so, whichever Z_{ij} line is followed to the apex. Clearly, any two of the Z_{ij} lines determine the x and y part of the solution. The third one may serve as a check. The value of z to complete the solution may be found graphically by stepping along any Z_{ij} line from the $z = 0$ point to (3, 4) using the distance between the points for $z = 0$ and $z = 1$ as the "unit length" for that line. Or $x = 3$ and $y = 4$ may be substituted in one of the equations (3a, 3b, or 3c) to find the z value. Either way $(x, y, z) = (3, 4, 2)$ is the complete solution.

Equations 4a, 4b, and 4c are plotted in figure 5 for $z = 0$ and $z = 1$ where the intersections of equations 4a and 4b are considered off the graph:

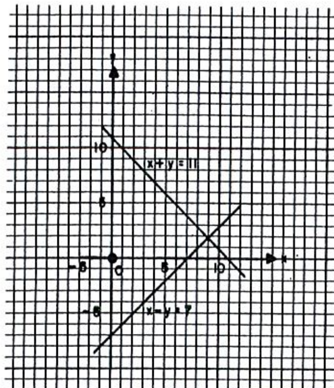

FIGURE 1

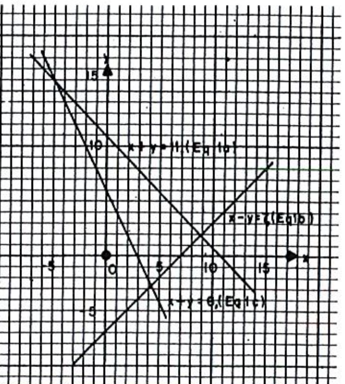

FIGURE 2

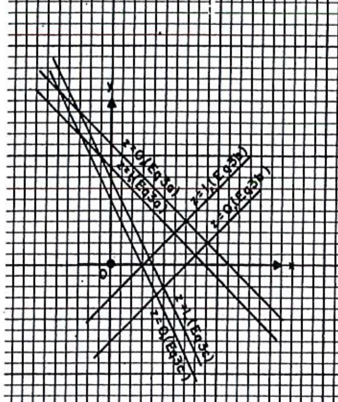

FIGURE 3

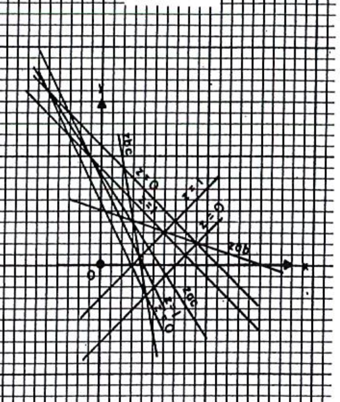

FIGURE 4

$$x + 8y + 24z = 54 \quad (4a)$$

$$x - 6y + 12z = 18 \quad (4b)$$

$$5x - y - 4z = 2 \quad (4c)$$

Although Z_{ab} cannot be established; Z_{ac} and Z_{bc} are effectively equations 4a, 4b, and 4c, with z eliminated. Their intersection produces the x and y values of $2\frac{1}{5}$ and 1 as read from the graph. The algebraic solution yields $x = 2\frac{180}{311}$, $y = 1\frac{1}{371}$ and $z = 1\frac{754}{933}$. In figure 5, the parallel lines for each of the equations 4a, 4b, and 4c are fairly close together. Instead of using 0 and 1 for z any other two values might have been used. Had 0 and 5 been used instead, the construction could have been done more accurately and the discrepancy for x between the graphic and algebraic answers would have been less.

Equation 5a has no z term; that is the coefficient of z is zero. This is like c_1 of equation 2a when plotted in figure 2.

$$5x + y = 10 \quad (5a)$$

$$x + 4Y + 19z = 25 \quad (5b)$$

$$5x - 3y - 20z = 33 \quad (5c)$$

Thus, any change in z will have no effect on the plot of equation 5a. One consequence of this is that Z_{ab} and Z_{ac} will coincide with the plot of equation 5a. Another consequence is that the solution will lie at the intersection of Z_{bc} and the line for equation 5a. Figure 6 is a plot of equations 5a, 5b, 5c for z = 0 and z = 1. Here, Z_{bc} is parallel to equation 5a so no solution exists. Quantity Z may be eliminated between equations 5b and 5c. If so, the result can be simplified to $5x + y = 49$ as the equation for Z_{bc}, confirming or denying the graphic solution.

Figure 7 is a plot of three linear equations for $z = z_1$ and $z = z_2$. Figure 7 shows the four possible locations of the solutions, so as not to reveal which lines are for Z_1. One possible solution lies inside the triangles for the original equations at (3, 6) while the other three possible solutions lie outside the triangles (-3, 11), (-1, -2) or (11, 8). Note particularly that three Z_{ij} lines meet at every possible solution and that the parallelograms made by the two lines of the original equations may generate either of two Z_{ij} lines.

What has been done thus far may be summarized as follows. Each of the three equations in three variables may be plotted on a graph for any two variables by considering the third variable as a constant. Plotting any equation for different values of the third variable produces sets of parallel lines. The intersection of the lines for any pair of equations plotted (with the third variable held at the same constant value in both equations) is a point at which both equations can be satisfied simultaneously for the third variable constant as selected. Two such points determine the line which represents the equation found by eliminating the third variable from that pair of equations. Finally, the intersection of two such lines from two different pairs of equations is the point at which the two variables of plotting will satisfy the three equations. The problem of finding the value of the third variable remains. This can be done as described for equations 3a, 3b, and 3c.

Referring to figure 5, note that plotting six lines and drawing two more gives the solution as far as x and y are involved. A total of eight lines were used for three variables. A similar degree of solution for four variables will require at least 20 lines, although more may sometimes be used to advantage.

Before looking at a numerical example, consider some of the parts of the problem. Assume there are four equations, a, b, c, and d, for which a solution exists, or which are not redundant as were equations 5a, 5b, 5c of figure 6, with the variables w, x, y, and z, and plotted in x and y. Then pairs of values for w and z must be selected, such as (w, z) = (o, o), (o, z_1) (w_1, o) etc. Plotting all four equations with (w, z) = (0, 0) will give some general idea of where the intersections will fall. Possibly,

FIGURE 5
FIGURE 6

FIGURE 7
FIGURE 8

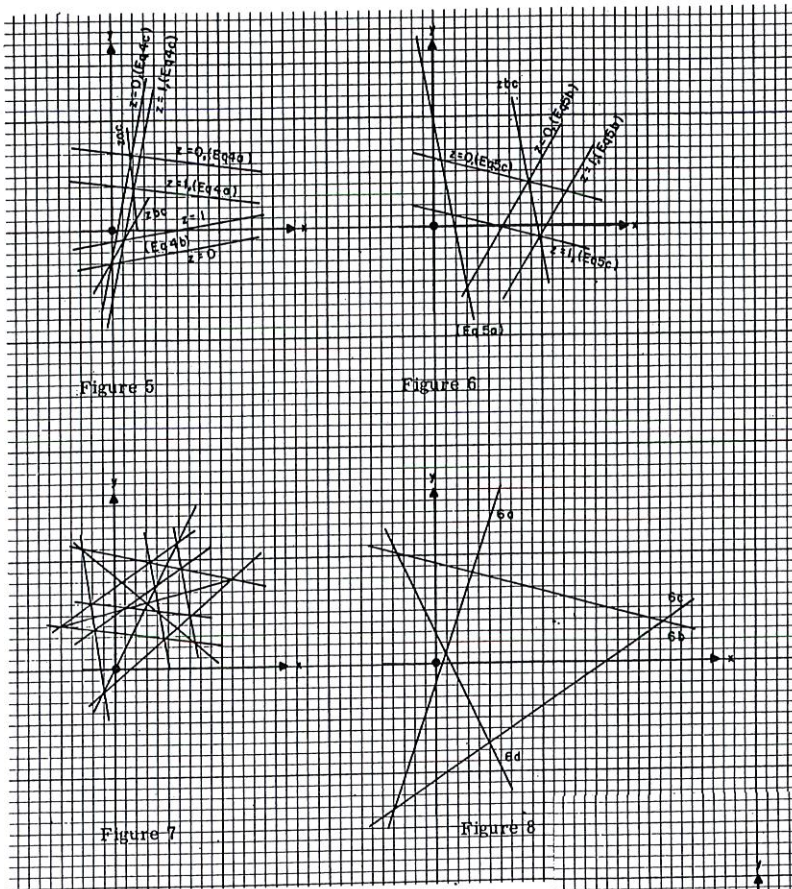

Figure 5

Figure 6

Figure 7

Figure 8

FIGURE 9
FIGURE 10
FIGURE 11

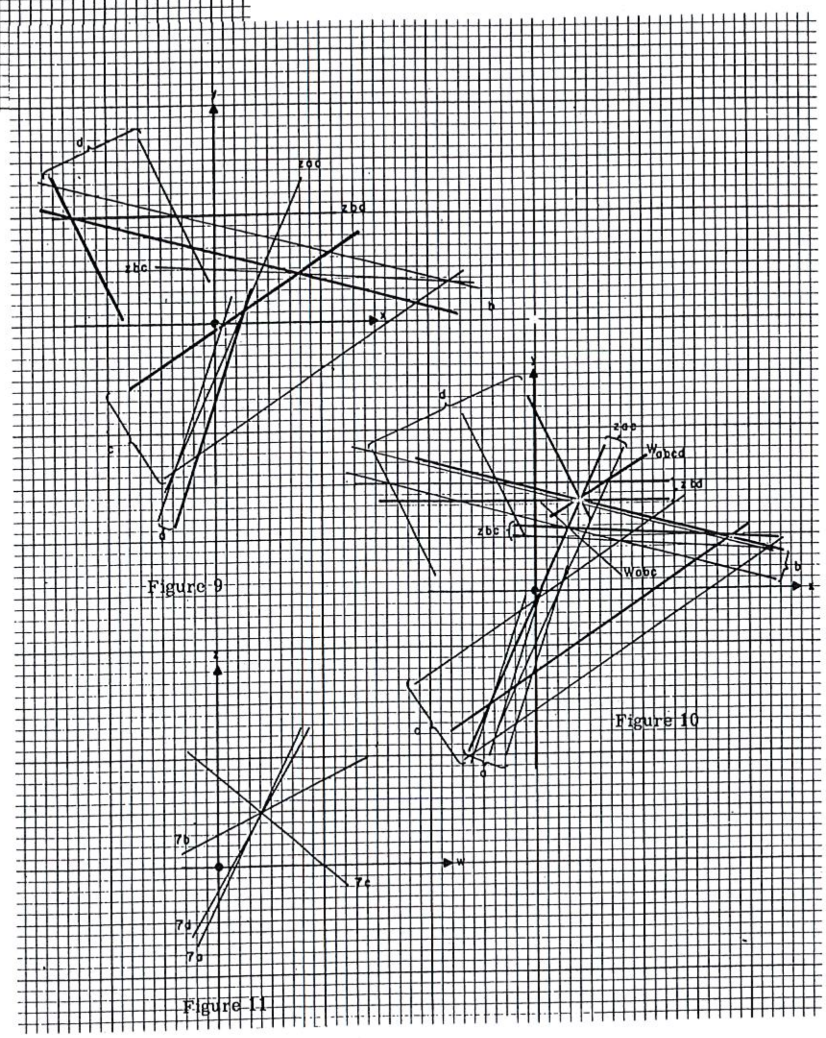

Figure 9

Figure 10

Figure 11

all six intersections of these four lines will be within the limits of the graph. Only three are needed — any three involve all four lines. Assume the intersections chosen are of equations a and b, of equations a and c, and of equations a and d. Now draw the lines of the four equations with $(w, z) = (o, z_1)$. Draw only enough of each line to establish the second set of intersections of equations a and b, of equations a and c, and of equations a and d; more will clutter the graph needlessly. Through the two intersections of each of the three sets now draw the Z_{ij} lines. These Z_{ij} lines are the graphs of equations in w, x, and y with w = o, which could be found algebraically by eliminating z from between the various pairs of equations a, b, c and d. Since the plots are of three variable equations, they correspond exactly to lines plotted from the original equations for three variables in earlier examples. Now plot the four equations with $(w, z) = (w_1, o)$. Again, only enough of each line should be drawn to establish the three intersections of interest. Through these points, lines parallel to the proper Z_{ij} lines can be drawn. These pairs of parallel lines correspond to the pairs plotted from equations in three variables. The solution may be completed as shown for three variables.

Equations 6a, 6b, 6c, and 6d will be used as a numerical example:

$$2w + 3x - y - z = 2 \quad (6a)$$

$$w - x - 4y - 2z = 36 \quad (6b)$$

$$5w + 2x - 3y + 6z = 33 \quad (6c)$$

$$-7w + 2x + y + 4z = 3 \quad (6d)$$

Figure 8 is a plot of equations 6 with (w, z) = (o, o). From this figure an arbitrary selection of the intersections a-c, b-c, and b-d is made. In figure 9 the selected part of the plot of these equations is made in fine lines for (w, z) = (o, o) and in bold lines for (w, z) = (o, 5); the three Z_{ij} lines are drawn also. In figure 10 all of figure 9 is repeated in fine lines and the bold lines are for equations 6a, 6b, 6c, and 6d with (w, z) = (2, o) and the new Z_{ij} lines. The symbol W with subscripts will be used to refer to lines constructed from the Z_{ij} lines in the same way the Z_{ij} lines are constructed from the lines of the original equations. The intersection of W_{abc} and W_{abcd} at (x, y) = (2, 7) is half the answer. Substituting these values in equations 6 and simplifying yields equations 7a, 7b, 7c, and 7d which are all plotted in figure 11:

$$2w - z = 3 \quad (7a)$$

$$w - 2z = -3 \quad (7b)$$

$$5w + 6z = 50 \quad (7c)$$

$$-7w + 4z = -8 \quad (7d)$$

All four lines intersect at (w, z) = (4, 5). The complete solution of equations 6a, 6b, 6c, and 6d is (w, x, y, z) = (4, 2, 7, 5). Note that using $z_1 = 5$ in the first stage of the solution did not reveal from the figures that z = 5 was the correct value for the solution. If (w, z) = (o, o) and (2, o) were used in plotting figure 9 then in figure 10 the heavy W_{ij} lines would all have met at (x, y) = (2, 7) indicating z = 5 as the correct answer.

SUMMARY

Any set of N linear equations in N variables for which a solution exists may be solved by plotting N sets of N-1 lines each. Using certain correctly chosen N-1 intersections of these lines, N-1 sets of N-2 lines each may be determined. This process may be repeated until only two sets of one line each exist. The intersection of these last two lines gives the values of the variables chosen for plotting. The N equations are thus satisfied simultaneously when the correct values of all other variables are also used. These known values are substituted in any N-2 of the original N equations. Simplifying them gives N-2 equations on which the above process may be repeated to determine the values of two more variables. This solving, substituting, simplifying, and resolving may be repeated until all variables are known.

SCIENCE IS WHERE YOU FIND IT

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Associate Prof. of Engineering Sciences,
Northwestern University

Science has been defined as knowledge amassed, severely tested, coordinated and systematized. Engineering education is increasingly science - oriented not only as concerns topical fields, but in essential character and mental discipline. The widely-held belief that a harvest of greater originality and productivity will result from the change also leads to a scrutiny and revision or reduction of courses irrelevant to or incompatible with the new goals.

Descriptive Geometry has the traditional charge of promoting spatial visualization; we propose to demonstrate that, in addition, it can effectively enhance scientific and analytic thinking.

The main requisite is an orderly structure of ideas and we start with an itemization of the minimum base assumed to be included in the students' preparation, as follows.

A. Concepts.
Point, line (meaning straight line), segment of a line, plane, angle, point on line, point on plane and line on plane.

B. Vocabulary.
Intersecting, parallel, perpendicular, horizontal, vertical, triangle, isosceles, equilateral, etc.

C. Relationships.

1. Two points determine a line; three non-colinear points determine a plane.
2. Coplanar lines are parallel or intersecting.
3. There is exactly one line that is parallel to any given line and on a given point.
4. If one of two parallel lines is perpendicular to a line or a plane, they both are.
5. Two lines perpendicular to the same plane are parallel.
6. A line perpendicular to a plane is perpendicular to every line on the plane.
7. A line perpendicular to each of two intersecting lines is perpendicular to the plane of the lines.
8. There is exactly one line that is perpendicular to a given plane and on a given point.
9. There is exactly one plane that is perpendicular to a given plane and on a given line on that plane.
10. A line perpendicular to a plane intersects it on exactly one point.
11. There is exactly one line that is perpendicular to and intersecting a given line and on a given point not on the given line.
12. If a line is perpendicular to a plane, any plane on the line is perpendicular to the plane.
13. Any points A_1, A_2, A_3 - - - A_n located on a line for which positive and negative directions have been assigned determine directed segments such that

$$A_1A_2 + A_2A_3 + - - - + A_nA_1 = 0$$

14. Pythagorean Theorem.
15. Congruency Theorems.
16. Properties of plane figures, their altitudes, areas, etc..

We add three definitions.

D1. Orthogonal Projection (verb) is a theoretical act of determining images of configurations by exclusively employing lines (called projectors) perpendicular to the reference plane or reference axis of any image determined; the points of intersection of any plane or axis and the related projectors comprise an image.

D2. Orthogonal projection (noun) is the image produced on a reference plane by orthogonal projection (verb).

D3. Orthogonal component (noun) is the image produced on a reference axis by orthogonal projection (verb).

Consider the usual x, y and z axes, mutually perpendicular in space, the xy, xz and yz planes determined by them and the z, y and x orthogonal projections on the planes, respectively. In each case the name of the projection appears as a subscript and denotes the direction of projection. See Figure 1. Segment AB has a_xb_x and a_zb_z as its projections on the yz and xy planes, respectively.

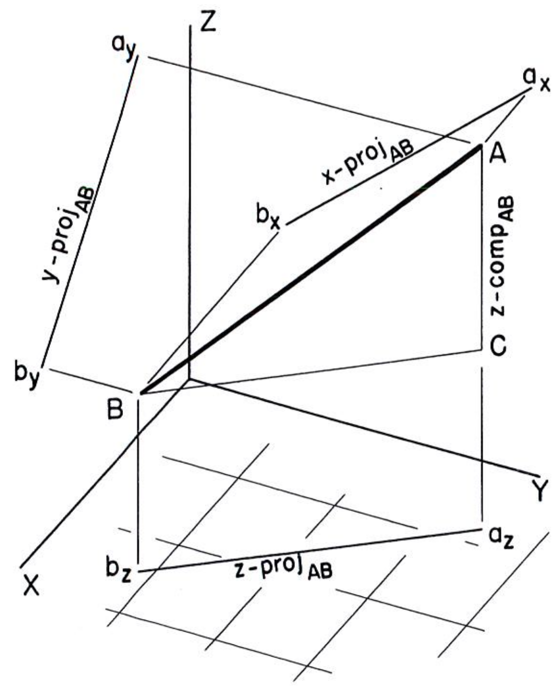


Figure 1.

It is easy to show that given any segment AB in space there is exactly one point C such that AC is parallel to the z axis and BC is parallel to the xy plane. (It is understood that AC and BC intersect at C.) ABC is a right triangle unless AB is parallel to the z axis or parallel to the xy plane. In any case, AC has length equal to the orthogonal component of AB on any line parallel to the z-axis; this is called the z-component of AB. Also, BC has length equal to the orthogonal projection of AB on any plane parallel to the xy plane; this is called the z-projection of AB.

For greater facility we disassemble Figure 1 and arrange the three planes together with the images cast upon them as shown in Figure 2. An important advantage to graphical analysis of this rearrangement is that any segment whose space position is parallel to one of the reference planes is registered actual or true length on that plane. This can be demonstrated to be a consequence of orthogonal projection. The positioning of the planes on the page is arbitrary, but in conformity with American Standard Practice for engineers.

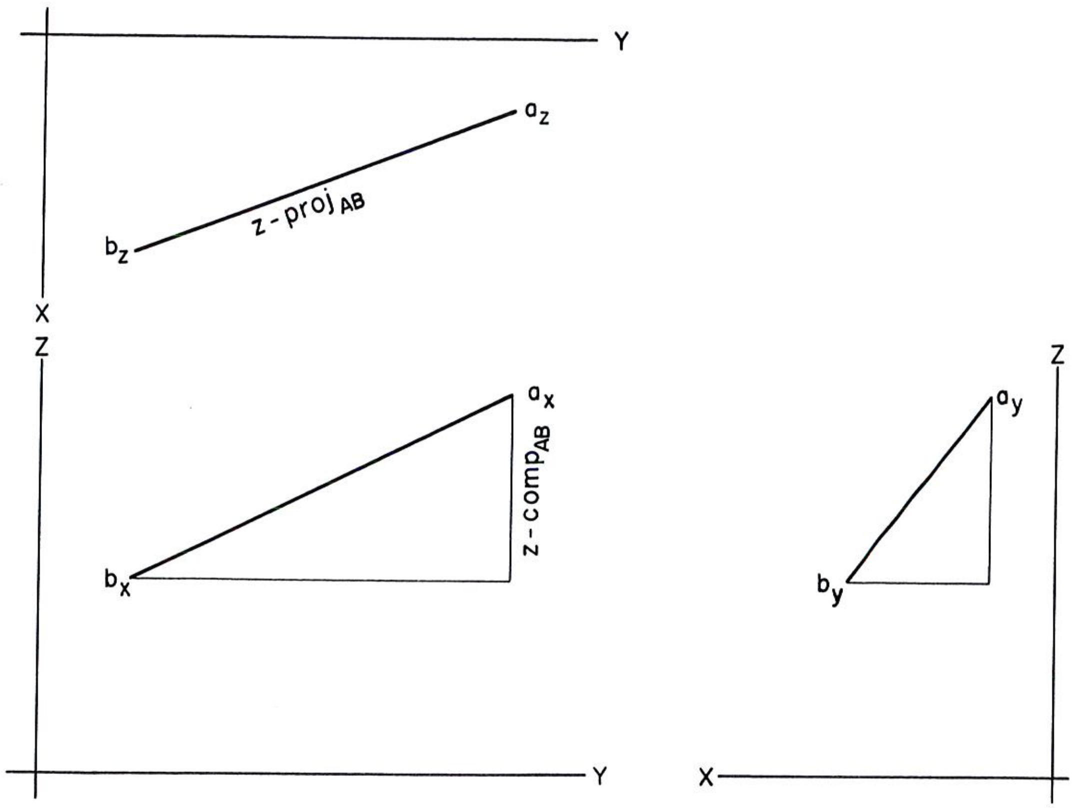


Figure 2.

Next, it is easy to determine the true length of any segment AB, regardless of space position provided it is mapped on the xy and yz planes. From the foregoing discussion we know that if AB is parallel to the z axis, $a_x b_x$ is the true length, if AB is parallel to the xy plane $a_z b_z$ is the true length, and in every other possible position there is a right triangle ABC. In this general case, we construct a triangle congruent to the space triangle ABC shown in distorted view in Figure 1, by using the length of the z-projection of AB as one leg, the length of the z-component of AB as the other leg perpendicular to the first. The true length is of the same magnitude as the hypotenuse. See Figure 3.

Add definition D4. The angle between any line and any plane is the angle between the line and its orthogonal projection on the plane. Thus angle ABC in Figure 3 records the angle AB forms with the xy plane.

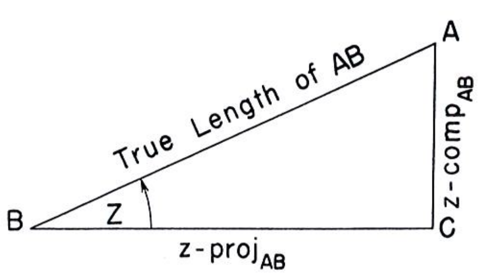


Figure 3.

For consistency of notation we shall call this the angle z of AB and Figure 3 the z-space triangle of AB.

It is clear that the z-space triangle contains four (4) items of information regarding segment AB, viz., true length, z-projection, z-component, and angle z. Any two of these items known or given determine the triangle and thus yield the other two.

It is also evident that triangles ABD and ABE lead to x- and y-space triangles as shown in Figure 4 and Figure 5, respectively.

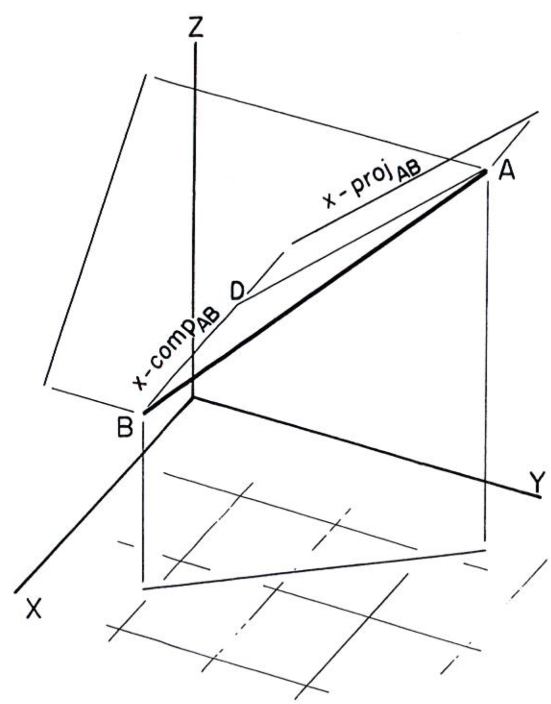


Figure 4.

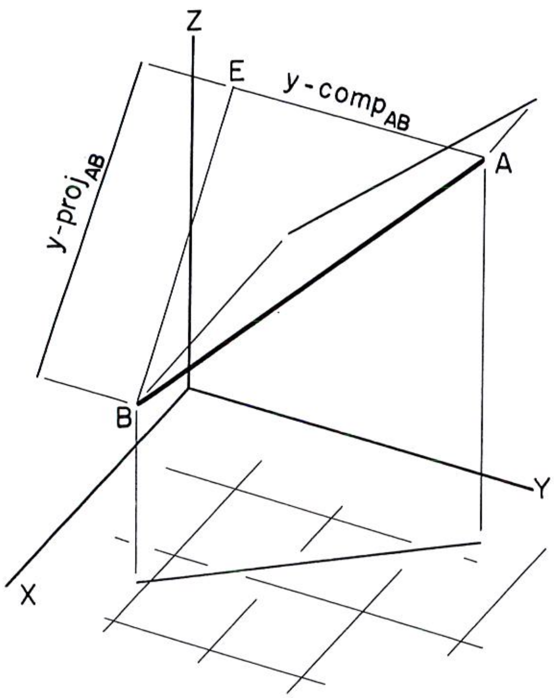
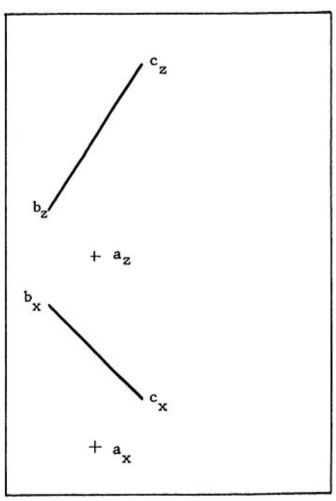


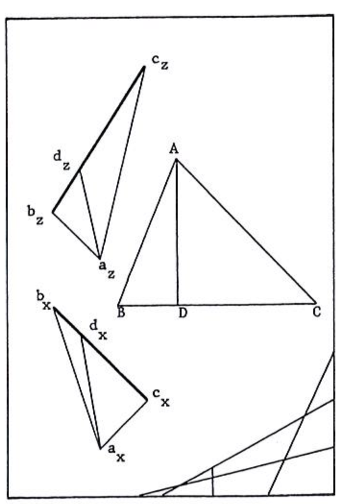
Figure 5.

There is no implication that the reader needed the foregoing discussion or that any course would be limited to it. The intention is to itemize a group of ideas and then present problems which are challenging educational experiences for students after such small (and then increasing) amounts of instruction.

PROBLEM 1 Find the distance from point A to line BC. Figure 6.



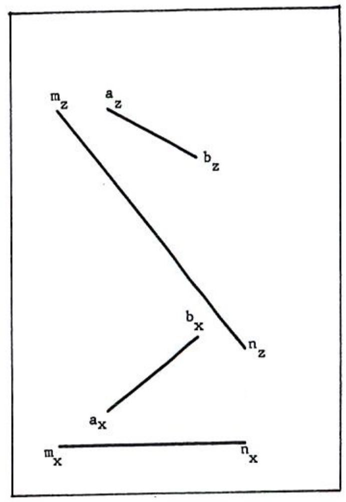
6 a
The Given Problem 1.



6 b
Solution of Problem 1.

After exploratory work with the representation of perpendicularity and intersection, which matters have not been discussed and are not amenable to "common sense" in this case, many students work with triangle ABC, find the true length of each of the sides AB, BC and CA, construct a true image of ABC using these lengths and then determine the length of BD, the altitude to BC.

PROBLEM 2 Construct the x and z maps of triangle ABC. Point C is on MN, MN is parallel to the xy plane and AB = AC. Figure 7.



7 a
The Given Problem 2.

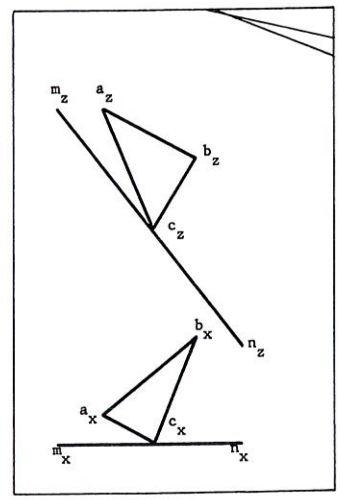


Figure 7 b
Solution of Problem 2.

The true length of AB is readily determined. This is also to be the true length of AC. From the fact that MN is parallel to the xy plane we see that the distance from a_x to $m_x n_x$ is the z- component of AC. We can now construct the z-space triangle of AC, determine the length of $a_z c_z$, locate c_z on $m_z n_z$ and complete the solution.

PROBLEM 3 Construct the x and z maps of the smallest possible equilateral triangle RST. Figure 8.

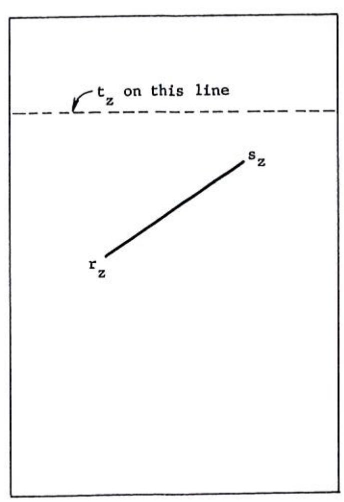


Figure 8 a
The Given Problem 3.

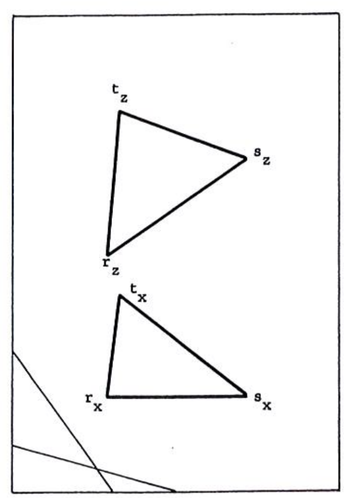


Figure 8 b
Solution of Problem 3.

We are given the z-projection of RS and know that

$$TL_{RS} = \sqrt{(z\text{-proj}_{RS})^2 + (z\text{-comp}_{RS})^2}.$$

Clearly the shortest true length occurs when the z-component of RS is zero. Thus the z projection of RS equals the true length, RS is parallel to the xy plane with $r_x s_x$ parallel to the y-axis in the x map and t_x located as follows. We now know the true length of RT and ST as they equal the true length of RS; we also know the x-component of RT and ST and we can construct the x-space triangles of RT and ST. From these we can obtain the x-projection of RT and the x-projection of ST, respectively, and are able to locate t_x.

We were given the x-coordinate of T and now have determined the y- and z-coordinates. Thus t_z is easily plotted.

It is an interesting exercise to prove

Theorem 1. If and only if one of two perpendicular lines is parallel to a plane, their projections on that plane are perpendicular.

Given perpendicular segments AB and MN, consider AB and BC with BC parallel to MN and intersecting AB. ABC is a right triangle and by C14,

$$(\text{True length of AB})^2+(\text{True length of BC})^2=(\text{True length of CA})^2.$$

Recalling the z-space triangles of AB, BC and CA, respectively,

$$(z\text{-proj}_{AB})^2+(z\text{-comp}_{AB})^2+(z\text{-proj}_{BC})^2+(z\text{-comp}_{BC})^2=(z\text{-proj}_{CA})^2+(z\text{-comp}_{BC})^2.$$

or

$$(z\text{-proj}_{AB})^2+(z\text{-proj}_{BC})^2=(z\text{-proj}_{CA})^2+(z\text{-comp}_{CA})^2-(z\text{-comp}_{AB})^2-(z\text{-comp}_{BC})^2.$$

It is clear that the z-projections of AB and BC are perpendicular if and only if,

$$(z\text{-comp}_{CA})^2-(z\text{-comp}_{AB})^2-(z\text{-comp}_{BC})^2 = 0.$$

Consider directed segments around the x-map of ABC, which is a closed polygon, and recall C13.

$$z\text{-comp}_{CA} + z\text{-comp}_{AB} + z\text{-comp}_{BC} = 0.$$

or

$$(z\text{-comp}_{CA})^2 = (z\text{-comp}_{AB})^2 + (z\text{-comp}_{BC}) + 2(z\text{-comp}_{AB})(z\text{-comp}_{BC}).$$

or

$$(z\text{-comp}_{CA})^2-(z\text{-comp}_{AB})^2-(z\text{-comp}_{BC})^2 = 2(z\text{-comp}_{AB})(z\text{-comp}_{BC}).$$

Thus, the z-projections of AB and BC are perpendicular if and only if,

$$(z\text{-comp}_{AB})\ (z\text{-comp}_{BC}) = 0.$$

requiring AB parallel to xy.

or BC parallel to xy.

PROBLEM 4 Construct CD perpendicular to AB. Figure 9.
Suggestion: Make reasonable assumptions regarding the given data.

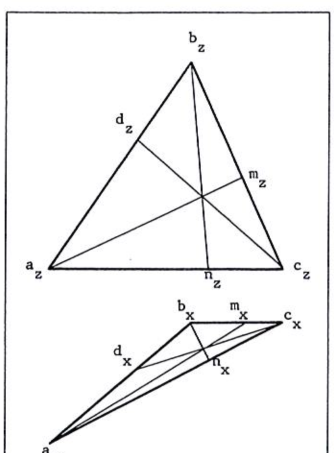


Figure 9

In Figure 9 it is clear that AB is not parallel to any of the three planes we have discussed, but it is reasonable to assume that AC is parallel to yz and that BC is parallel to xy. Thus, we can construct $a_z m_z$ perpendicular to $b_z c_z$, locate m_x on $b_x c_x$ and draw $a_x m_x$. Similarly, we can construct $b_x n_x$ perpendicular to $a_x c_x$, find n_z on $a_z c_z$ and draw $b_z n_z$. As the three altitudes of a triangle are concurrent (C16), we can now draw $c_z d_z$ and $c_x d_x$.

PROBLEM 5 Complete necessary views of isosceles right triangle ABC. Figure 10.

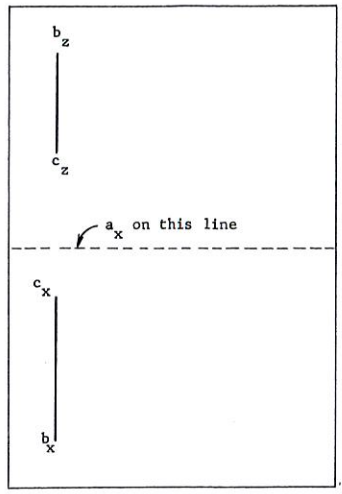


Figure 10 a
The Given Problem 5.

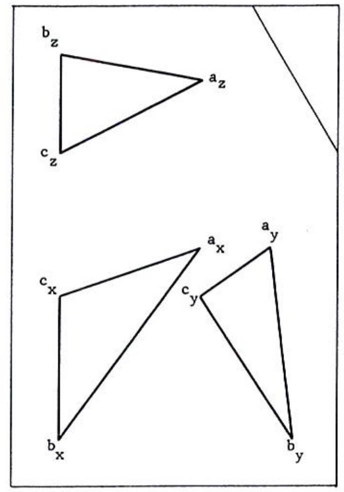


Figure 10 b
Solution of Problem 5.

The reasonable assumption that BC is parallel to xz, leads us to find $b_y c_y$ and construct $c_y a_y$ perpendicular to $b_y c_y$ with a_y at the same z-position as a_x. Clearly, the two legs of an isosceles right triangle have the same length and $b_y c_y$ records the true length of BC. As we have the y-projection of AC ($c_y a_y$) and the true length of AC we construct the y- component of AC, and thus we locate a_x and a_z.

Various engineering situations can be handled more conveniently if we assign particular space positions to the x, y and z axes. For example, we shall consider the x-axis to be horizontal with the positive portion south of the origin and the negative portion north. The y-axis shall be considered horizontal with the positive portion east of the origin and the negative portion, west. The positive z-axis is considered up from the origin and the negative portion, down. Now we see that the xy plane or any line parallel to it is horizontal, the xz plane is in a vertical north-south position and the yz plane is in a vertical east-west position.

We add the following definitions:

D5. The bearing of a line is the compass direction of its z map; it is described with a three-part symbol; N or S, an angle θ, $0^o < \theta < 90^o$, and E or W. Thus 'AB bears N30°W' means that $a_z b_z$ makes a 30° angle with the x-axis and b_z is north and west of a_z.

D6. A strike line of a plane is any line that is on the plane and parallel to the xy plane.

D7. The strike of a plane is the bearing of any strike line of the plane

D8. The dip of a plane is the angle z of any line that is on the plane and perpendicular to a strike line.

D9. The angle between two lines, two planes or a line and a plane is always less than or equal to 90°.

D10. The angle between two planes is the angle between two lines, one in each plane and both perpendicular to the line of intersection of the planes.

The following demonstrations are interesting and useful.

Lemma 1: The dip of a plane equals the angle the plane forms with the xy plane.

Figure 11 illustrates the xy plane, a general plane W and their line of intersection, SN. RM is a strike line of W, RS is perpendicular to RM, and TS is the z-projection of RS. By D4 and D8 the true size of angle RST is the dip of plane W, and RT is perpendicular to the xy plane. TR is perpendicular to SN by C6. RM and SN are parallel as they are coplanar and not intersecting (C2) and SN is perpendicular to RS by C4. Thus SN is perpendicular to plane SRT (C7) and also perpendicular to ST (C6). The true angle RST equals the angle plane W forms with the xy plane by D10.

Lemma 2: A line perpendicular to a plane with a dip-angle of θ° has a z-angle equal to $90^{\circ}-\theta^{\circ}$.

In Figure 11, ER is perpendicular to W and plane ERS is perpendicular to W. Also, plane ERS coincides with plane TRS by C9. Angle RET represents the z- angle of ER by D4 and RST represents the dip angle of W. As angle ERS is a right triangle $\angle RET = 90^{\circ} - \angle RST$.

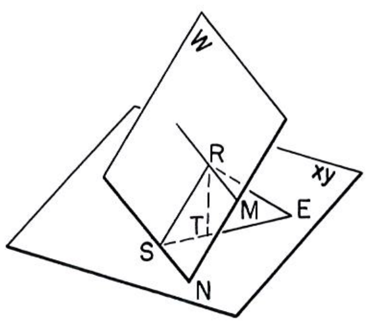


Figure 11

By generalizing lemma 2 we obtain:

Theorem 2. If a line and a plane are perpendicular they form complementary angles with any other plane.

PROBLEM 6 Represent isosceles right triangle ABC with side BC horizontal and hypotenuse AB bearing N75°E. Figure 12.

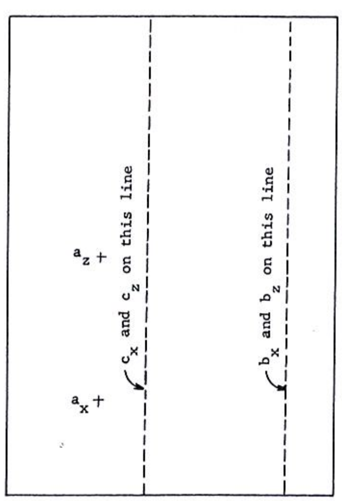


Figure 12 a
The Given Problem 6.

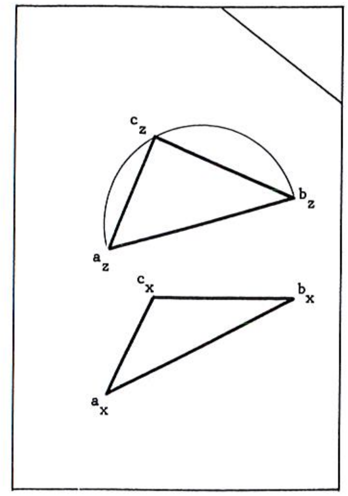


Figure 12 b
Solution of Problem 6.

The given information enables us to represent $a_z b_z$. As BC is required to be horizontal angle $b_z c_z a_z = 90^{\circ}$, and C_z is located on a semi-circle with $a_z b_z$ as diameter. (There are two possible semi-circles and Figure 12 only shows one.) Now the true length of BC = $b_z c_z$; this must also be the true length of AC. A z-space triangle uses the length of $a_z c_z$ and TL_{AC} to determine z-comp$_{AC}$.

PROBLEM 7 Represent a plane ABC that has strike of S30°W and forms a 50° angle with the xz plane. Figure 13.

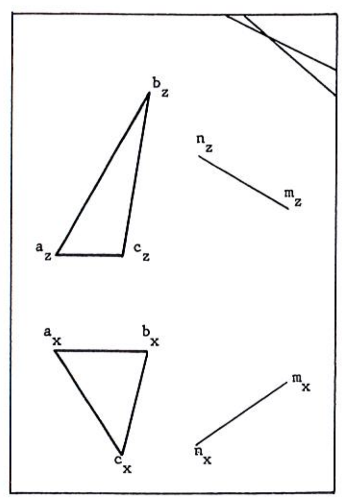
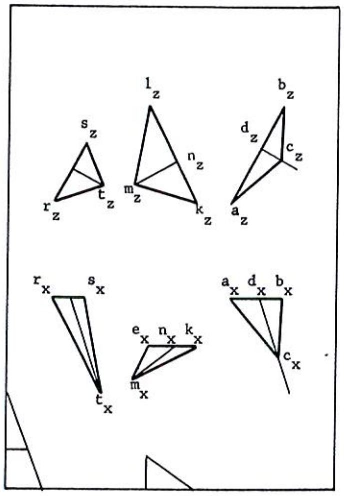


Figure 13

AB can be chosen as the strike line and it can be represented any desired length, horizontal and with bearing of S30°W. Any line, say MN, that is perpendicular to plane ABC must have angle y equal to 40° and have $m_z n_z$ perpendicular to $a_z b_z$. MN can be represented any desired length, by using a y-space triangle. AC, any desired length, can be represented if we choose to have it parallel to the yz plane for then $a_x c_x$ must be perpendicular to $m_x n_x$.

PROBLEM 8 Represent triangle ABC that is congruent to KLM and parallel to immovable plane RST. Figure 14.



Conveniently, both KLM and RST have a horizontal side or strike line as given. Then, a_zb_z is drawn parallel to r_zs_z and equal to k_zl_z. Next, represent the dip lines of RST and KLM, respectively; the dip line of ABC must be parallel to that of RST and start from d_z on a_zb_z, where $a_zd_z = l_zn_z$. Finally, set off a segment equal in length to MN on the dip line of ABC to find c_z and c_x.

The foregoing material illustrates in part a presentation of descriptive geometry consistent with today's science-oriented objectives. The problems have been used in homework assignments or examinations and produced an acceptable distribution of achievement plus considerable interest. The continuing invention of situations exploiting new combinations of the basic core of relationships is challenging and rewarding.

PROPOSED CHANGES IN CONSTITUTION AND BY-LAWS OF THE DIVISION OF ENGINEERING GRAPHICS OF ASEE

The following changes are recommended to bring out By-Laws into conformity with the By-Laws of the ASEE Council of Technical Divisions and Committees of which the Engineering Graphics Division is a member.

OLD:

Article II. PURPOSES. The purposes of the Division are identical with those stated in Article I, Section 3, of the Constitution of ASEE with special emphasis on those objectives as they pertain to the field of Engineering Drawing, Descriptive Geometry, and related subjects.

NEW:

Article II. OBJECTIVES. The Division shall coordinate and promote the interests and activities that pertain to the field of Engineering Graphics both within their own group and in the Council of Technical Divisions and Committees of ASEE.

OLD:

Article IV. EXECUTIVE COMMITTEE. The affairs of the Division shall be administered by an Executive Committee consisting of:

. . .

5. Immediate Past-Chairman of the Division, -member of the Executive Committee until the term of office of his successor as Chairman has been completed.

NEW:

Article IV. EXECUTIVE COMMITTEE. The affairs of the Division shall be administered by an Executive Committee consisting of:

. . .

5. Immediate Past-Chairman of the Division, -member of the Executive Committee until the term of office of his successor as Chairman has been completed and member of the Executive Board of the ASEE Council of Technical Divisions and Committees.

DELETE:

Article IV. . . .

9. Representative on General Council of the Society-elected in even numbered years for a term of two years.

NEW:

Article V. DUTIES OF OFFICERS. . . .

1. Chairman a) (no change)
   b) He shall be a member of the Executive Board of the ASEE Council of Technical Divisions and Committees.

CHANGE:

Old items b), c), d), e) to c), d), e), f) respectively. (No change in wording)

Article V. DUTIES OF OFFICERS

OLD: . . .

2. Vice-Chairman a) (no change)
   b) He shall preside over business meetings of the Division and Executive Committees in the absence of the Chairman.

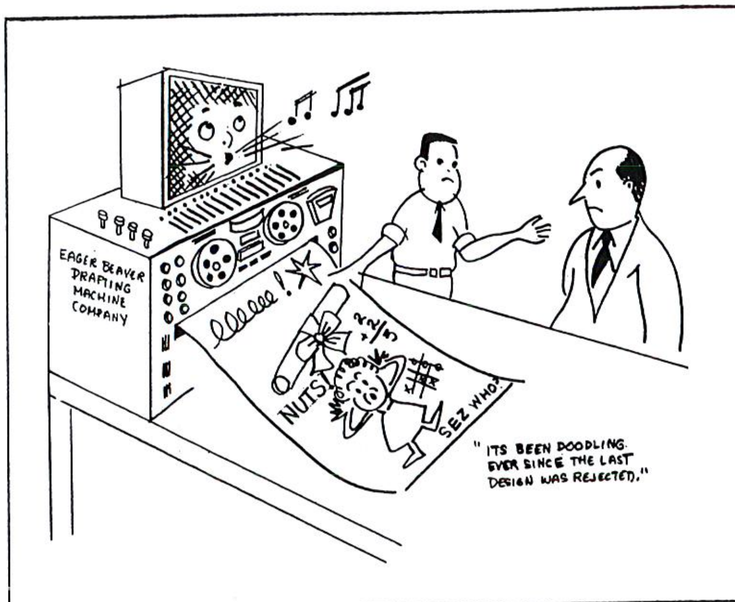
NEW:

b) He shall preside over business meetings of the Division and Executive Committees in the absence of the Chairman. Also, he shall represent the Chairman on the Executive Board of the ASEE Council of Technical Divisions and Committees should the Chairman of the Division be unable to act or should request the substitute representation.

Respectfully recommended
by the Policy Committee,
Engineering Graphics Division,
ASEE,

J. Gerardi
F. A. Heacock
R. P. Hoelscher
W. E. Street
J. S. Rising, Chairman

"Machine-Aided Graphics"
By
Kenneth E. Lofgren



Professor Kenneth E. Lofgren has been dreaming of the universal machine which can design a perpetual motion machine despite the first, second and third laws. Unfortunately he caught the machine doodling!

News of the Division of Engineering Graphics
1963 Midwinter Meeting. Manhattan Kansas
Kansas State University. January 23-25, 1963
Highlights: "The general prospects of Engineering Graphics as they relate to the changing pattern of"Engineering Education" by Newman Hall, Executive Director, Commission on Engineering Education.

Secretary of the Division

The candidate for Secretary of the division replacing nominee Robert La Rue will be Professor Stuart C. Allen, Michigan School of Mines, Houghton, Michigan.

Distinguished Service Award for 1962-63

Irwin Wladaver (New York) Chairman of the Special Awards Committee would like to receive your suggestions of the next recipient.

The criteria established by the Division are that the recipient of the Distinguished Service Award should be outstanding in:

1. Success as a teacher and ability to inspire students
2. Improvement in the tools and conditions for teaching
3. Improvement of teaching through various activities
4. Scholarly contribution
5. Service to the Division of Engineering Graphics

NSF Study on Engineering Graphics Course Content Development

The study of Engineering Graphics in the engineering colleges of the country which was begun over a year ago is entering a report writing and concluding stage. Chairman Paul Reinhard of the University of Detroit has organized studies, discussions, reviews and problem groups all over the country. Preliminary conclusions will be published in the May '63 Journal.

Publications Policy Committee

The A.S.E.E. has established a Publications Policy Committee. The Editor of the Journal of Engineering Graphics has been appointed a member.

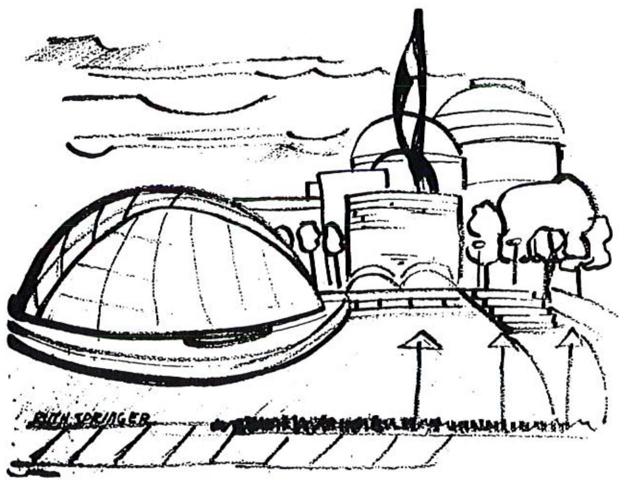
Courses for High School Drawing Teachers

Kraus Kroner of the University of Massachusetts has been interested for several years in the improvement of teaching in mechanical drawing courses in high schools. He has organized a 1963 summer course for high school teachers. This is the first such course in the North Eastern areas. For detailed information write to Professor Kroner at University of Massachusetts,

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

BY

RUTH SPRINGER



Above is a sketch of MIT by Mrs. Ruth Springer. In August 1962, the MIT conference on Design enabled Professor Robert Mann to air his views concerning engineering education. He made the principal address at the Middle-Atlantic section meeting at Stevens Institute in December. He will speak again at the forthcoming ASEE annual meeting in Philadelphia June 16-23, so that all Graphics Division members may hear what imaginative programs are in progress at MIT.

Amherst, Massachusetts. The course content is described as follows:

This course is being offered in anticipation of a nationwide trend on the part of engineering colleges to urge standardization of the pre-engineering drawing programs in the high schools. The course is open to high school drawing teachers and to those who contemplate teaching this subject.

The purpose of the course is to acquaint the teachers with the individual topics of mechanical drawing in which students entering engineering colleges would be expected to have a certain degree of proficiency, enabling them to proceed quickly to college-level work in the engineering graphics curricula. Attention will also be drawn to industry's expectations of the graduate engineer in respect to his ability to communicate graphically, and to the latest developments in drafting procedures, standards, and reproduction methods. For portions of this phase of the course, guest lectures from industry will be invited.

The lectures pertaining to individual graphics topics will include a review of theories of projections and will be supplemented by practical applications on the drawing tables of the engineering graphics laboratory.

NSF International Conference on Space Geometry

This proposed conference which was sponsored by Steve Slaby at Princeton University was turned down by NSF. This is a keen disappointment as members of the Division have been most eager to exchange ideas with foreign graphics teachers and researchers.

We hope that Professor Slaby will be able to secure funds for this important conference for the year 1964.

The Division regrets the recent death of one of its most active and loyal members

ORRIN W. POTTOR

He was Professor Emeritus of Engineering Graphics, At The University Of Minnesota. Professor Pottor was also a Life Member of the ASEE, which he joined thirty-seven years ago.

NATIONAL GRAPHICS STUDY IN PROGRESS

A national study of engineering graphics content begun in November of 1962 is currently in progress. This project is being conducted by Ernest C. Schamehorn of West Virginia Institute of Technology and is a questionnaire study on the opinions of engineering graphics educators, engineering educators of degree-granting departments, and practicing engineers concerning the content of engineering graphics courses. Participants are being asked to rate the degree of emphasis which they feel should be given to various topics, and to indicate how much time should be spent on the basic presentation of these topics. Five groups of engineering educators are included: (1) Engineering Graphics, (2) Mechanical, (3) Civil, (4) Electrical, (5) Chemical. The colleges and universities selected for the study include one or more of the leading schools in each state having at least one engineering curriculum ECPD accredited. As of January 20th, there has been a 77% return from the engineering educators.

Also selected for the survey are engineers employed by both large and small industries representing very diverse products, such as aircraft, missiles, electronics, nuclear engineering, soap, tires, communications, textiles, oil, mining, etc. Engineers from four functional areas of activity are included: (1) Manufacturing and Production, (2) Research and Development, (3) Operations and Maintenance, and (4) Architectural and Structural. A 52% return from the practicing engineers has been obtained to date, but this percentage is expected to increase significantly in the next few weeks.

A subsequent report of the findings and recommendations of this study will be prepared for publication in the Journal of Engineering Graphics and Journal of Engineering Education.

ANNUAL MIDWINTER MEETING

JANUARY 23-24-25, 1963

KANSAS STATE UNIVERSITY

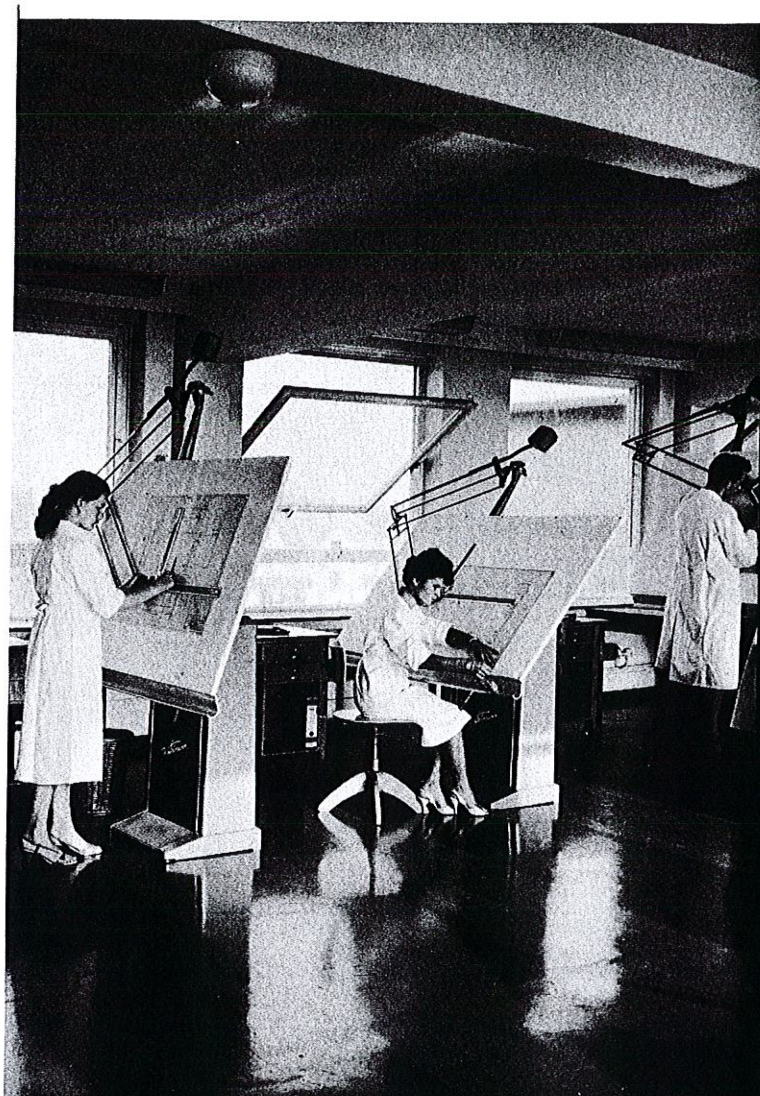
MANHATTAN, KANSAS

American Society for *Engineering Education*

DIVISION OF ENGINEERING GRAPHICS

ROSTER OF ATTENDANCE

Aakhus, T. T.	University of Nebraska
Ackert, Hugh P.	University of Notre Dame
Almfeldt, M. W.	Iowa State University
Arnbal, Carl A.	Iowa State University
Arnold, J. Norman	Purdue University
Atherly, Bill	Wichita University
Baer, Charles	University of Kansas
Bakken, Stewart E.	North Dakota State Univ
Banker, Deane	Hutchinson Junior College
Bates, H. T.	Kansas State University
Bell, Charles L.	Kansas State Teachers Col
Bennett, J. L.	Kansas City Junior Col
Besterfield, Dale H.	Southern Illinois Univ
Blade, Mary	Cooper Union
Black, Earl D.	General Motors Institute
Blevins, Craig M.	Missouri School of Mines
Botkin, K. E.	Purdue University
Brainard, B. B.	Kansas State University
Buck, Carson P.	Syracuse University
Burnett, James R.	Michigan State University
Burns, Frank	Newark College of Eng
Butler, Robert O.	Iowa State University
Call, W. R.	Allis Chalmers
Casey, Frank J.	University of Colorado
Cates, S. L.	Amarillo College
Christianson, L. C.	Missouri School of Mines
Coatney, Robert	K & E Company
Coke, John M.	Colorado School of Mines
Crawford, Joe V.	Iowa State University
Dahms, A. A.	Allis Chalmers
DeVaney, Amogene F.	Amarillo College
Dobbins, George S.	University of Colorado
Duncan, Alley	Kansas State University
Durland, M. A.	Kansas State University
Earl, William A.	Alfred University
Fallon, Richard T.	JETS, Inc.
Flinner, A. O.	Kansas State University
Gaskell, Alfred S.	University of Missouri
Gilliland, Hugh R.	Missouri School of Mines
Gohman, Richard W.	Iowa State University
Gowdy, Ken	Kansas State University
Grubb, Grover C.	Arlington State College
Hall, Clarence E.	Day & Zimmermann, Inc.
Hall, Newman	Exec. Dir. Comm. on Engg. Educ.
Hammond, Robert H.	U. S. M. A.
Higdon, Archie	U. S. A. F. Academy
Holmen, H. E.	Waldorf College
Honstead, A. E.	Kansas State University
Jacobs, R. Kenneth	Georgia Institute of Tech
Jannsen, David A.	Portland State College
Jenkins, Herbert T.	University of Michigan
Jones, Gordon L.	Hutchinson Junior College
Keith, Charles W.	Kent State University
Klopp, Mark E.	Southern Illinois Univ
Knox, C. S.	Allis Chalmers
Lake, Donald W.	Coffeyville College
Lindholm, John C.	Kansas State University
Long, T. R.	West Virginia University
Marion, Cecil P., Jr.	University of Miami
Messenheimer, A. E.	Kansas State University
Mochel, Myron G.	Clarkson College of Tech
McClain, Raymond G.	University of Missouri
McDonald, A. P.	Rice University
McNeary, Matthew	University of Maine
Neely, H. M.	Kansas State University
Nevins, Ralph	Kansas State University
Northrup, Ralph T.	Wayne State University
Oppenheimer, Frank	Gramercy Guild Group
Palmerlee, Al	University of Kansas
Pauli, Ross	Kansas State University
Perquin, Nicholas	Catholic University, Wash
Pfeffer, Charles B.	Wichita University
Philby, A. J.	Ohio State University
Pollock, Allen E.	Kansas City Junior College
Potts, Philip O.	University of Michigan
Reinhard, Paul M.	University of Detroit
Reynolds, R. Wallace	California State Poly
Richard, Sidney	Amarillo College
Ring, Ralph	University of Kansas
Rising, Jim	Iowa State University
Risser, Arthur C.	Wichita University
Roark, Ivan W.	Tulsa University
Sanders, Gordon C.	Iowa State University
Schamehorn, Ernest C.	West Virginia Institute of Tech
Schruben, L. R.	University of Southern Cal
Shupe, John	Kansas State University
Smith, Frank E.	Wichita University
Smith, Thomas C.	University of Nebraska
Smutz, F. A.	Kansas State University
Street, W. E,	Texas A & M College
Taylor, Kenneth O.	Consulting Engineer, Wichita
Teeters, Joseph L.	Baker University
Tripp, Wilson	Kansas State University
Walters, Howard R.	Unive rsity of Nebraska
Weidhaas, Ernest R.	Pennsylvania State University
Wellman, B. Leighton	Worcester Polytechnic Institute
Wladaver, Irwin	New York University
Wood, Joe N.	Kansas State University
Fetters, Bill	Boeing Aircraft Corporation,



Dear Mary:

Here are Jack Rule's comments on the parallel-axis article, which comments I wish had taken effect. It's my fault for not getting them to you.

Regards,

Forrest Woodworth
College of Engineering
University of Detroit

Some comments from Prof. John T. Rule of MIT concerning Forrest Woodworth's article in the Nov. 1962 Journal on "Parallel-Axis Projection".

---

1) Page 2 after Theorem II the sentence, "Granted the question is academic because applications of Projective Geometry are not ordinarily concerned with such things, ..." would be much better if it read, "Granted the question is a metric question and thus outside the realm of Projective Geometry which is basically non-metric, ..."

2) Page 3, "Note that these three points all lie on the same straight line! ... " would be better if it read, "Note that these three points must lie on the same straight line since they are the intersections of the sides of triangles whose vertices lie on concurrent lines (the parallel axis). ..."

Also Fig. 6 is unfortunate in that the collinear points (z,x), (x,y), (y,z) establish a vertical line (or almost so). This is an accident, not a necessity, but subjectively it caused me to spend a minute or so determining if, in fact, the line must be vertical.

Dear Mary,

Just received the November issue. Congratulations - it looks very good. I like the new title but would predict that you're going to have some squawks from certain quarters. Also, you might give your proof-reader H___. I detected errors in at least three articles.

Editor's note - see editorial - this issue.

Best regards,

Robert D. LaRue
Associate Professor
Colorado State University

Dear Professor:

I have just finished reading and enjoying the new issue of the Journal which came yesterday, (February 1962). I would like to make a few comments. The Shock Wave article is very good, but many of the other articles seem to reflect the same idea; that the new Graphics courses are not exactly perfect. How can we expect them to be, when they are required courses forced on freshmen who barely have their feet on the ground yet? As soon as descrip. is finished they forget all they have learned, and no instructor requires them to use anything they may have learned. Every later instructor should require good instrument-made drawings for any diagrams turned in for his course, and should not be satisfied with lettering like this: Robert W. Smith

Personally, I think that graphical calculus and curve plotting belong in the math department, with good instrument drawings required. Perhaps the drawing teachers could help.

Mechanical engineers, at least, need an advanced drawing course in the last year, to bring back what they learned as freshmen, and apply it with what they have since learned. The student's drawing on page 19 is very good, but how much better it would be to design and draw this as a college senior. His Woodruff key is wrong, and something looks wrong with the thrust bearing.

Here we can't put in much graphics but I have a couple of inovations you might find interesting. In place of the usual problem in descrip about a cylinder on an oblique center line, I have them draw a simple rocket - a cylinder with a pointed nose and four fins plus an exhaust. I supply the top and front

[redacted]

center lines, turned so that isometric templates can be used for the ellipses in both views. They find this quite interesting and I make it a bit harder by requiring that two fins be horizontal. (I'll stick one in if I can find one). Next, after they have finished the problems in descrip. about the angle between two planes, I give as an end-of-term project a working drawing of an oblique bracket (it is prob. 6.3.3 in French & Vierck). Most of them look at me in complete amazement - they have no inkling of the connection between descrip and the working drawings they made in the previous term. I practically have to show them how, but believe they profit by doing it. I require a partial view, an edge view, and one complete view. This last requires ellipses, and I let them use my 45° ellipse template.
At least two boys have said that employers are impressed with this sort of work.

So much for some random thoughts.

H. W. BLAKESLEE
Ass't. Prof. M.E.
Captain USN (Ret)
Central Florida Junior College

Dear Editor;

I am so sorry to have "flunked" graphics, but I am happy to be considered a professional engineer who solves a problem the easy way.

I would like to make a motion here and now to the effect that "Graphic Tantalizers" shall be stricken from all Graphics publications now and forever more; and hope that some kind soul will second this motion so that the majority (I think) will vote "AH".

Would it not be more to our mutual advantage and happiness to publish "Analytical Tantalizers" so that we may prove to the academic geniuses how easily they may be solved with the aid of graphics? An example would be non-coplaner forces. How easily (mathematical and algebraic) errors creep into the analytical solution, and how time consuming it proves to be. Yet, by application of the art of graphics, this very same probelm may be solved in less time than it is taking me to write this letter.

I do think mechanical drawing, oh, I'm sorry, I mean Engineering Graphics and Design, the real language of the engineer has suffered enough of a set back without further attempting to prove that graphics may be tantalizing.

Please, please let us prove to the academic geniuses that engineering drawing or graphics as a universal language is so essential to engineering design and that because they can't understand it is no valid reason to eliminate it from the curriculum.

Do you get my message?

As one engineer to another graphician would say -

Graphically & Tantalizing yours,

Irving Gorden
Prof. Engineer, N.Y.C.

EDITOR'S NOTE

Mr. Gordon solved one of the graphic tantalizers in the last issue by analytic means and our rejection of his solution has evidently drawn blood.

Dear Prof. Blade:

Are you aware that the Journal is received by me two weeks after your dead-line? I like your effort and the improvement. The Graphic Division is playing ostrich and is refusing to acknowledge disaster.

Sincerely,

John Barylsky

New Bedford Institute of Technology

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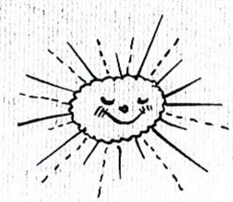
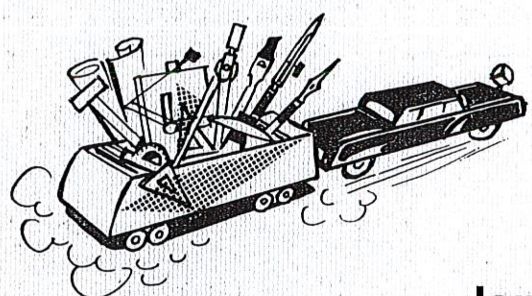




Our adress, January 1, 1963:

Gramercy Guild Group, Inc.,

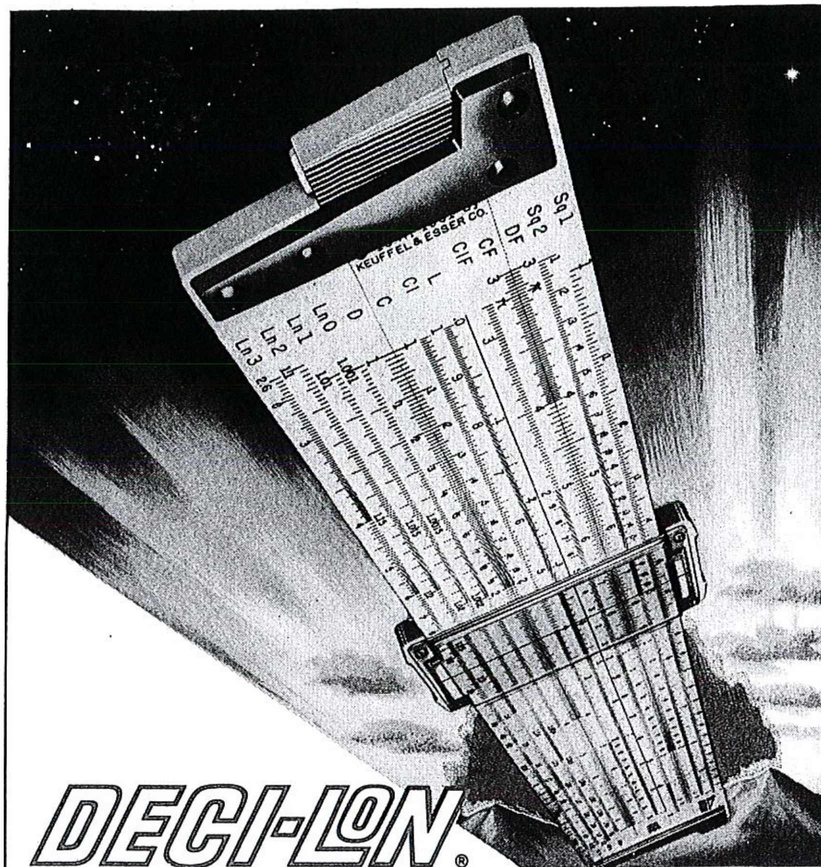
846 Elati Street, Denver 4, Colorado, Tel. Keystone 4-4251

In order to serve our customers better, we are moving the Gramercy headquarters, January 1st, 1963, to Denver, Colorado. In addition to our main warehousing operation in Denver, we have set up various localized distribution centers throughout the whole country. Wherever drafting is taught, and wherever drawing materials are used, in the Fifty States of the Union, Frank Oppenheimer, as always, will be completely available to discuss and aid in the solving of individual needs and problems. We are looking forward to settling in the beautiful Colorado mountains which are so close in character to the Bavarian Alps where the famous RIEFLER drawing instruments are made.

Our headquarters in Denver will be located at the Engineering Drafting School, where Miss Flora Dee Goforth, its brilliant director, has guided the development of this most unusual institution to its high standard of achievement. We are delighted that we will be in such close active contact with the TECHNICAL EDUCATION FIELD. If during the last two weeks of December, you find that we are not giving you good service, please forgive us and bear with us. You all know what moving is! However, we are planning and striving to keep the disruption at a minimum.

Printed in Germany

what we said...



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