

## Concurrent Coplanar and Non-planar Structural Analysis Using 3D CAD

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### ABSTRACT

*The purpose of this study was to investigate the effectiveness of 3D CAD in dealing with the concurrent coplanar and non-coplanar structural analysis. Several problems were solved using both 3D CAD and traditional approaches. The traditional approach in descriptive geometry requires manual construction of vector polygons. The 3D CAD approach, which utilizes the same geometric and spatial reasoning, requires constrained wireframes and extruded solids as the tools for analysis. The comparison of these two different approaches was made. It proved that 3D CAD approach did demonstrate (1) the high performance in speed and accuracy and (2) the effectiveness in examining the design alternatives.*

### Introduction

Much engineering data is graphical in nature. Graphical methods utilizing such data can provide comparable accuracy for analysis of vectors, such as forces, velocities and accelerations, in engineering mechanics, machine design and structural analysis (Pare, Loving, Hill & Pare, 1987). When two or more vectors act on an object through a common point, they are called concurrent vectors. It is often necessary in engineering practice and design work to resolve a known vector into concurrent coplanar or non-coplanar components. In descriptive geometry, this is usually handled with graphical construction of vector polygons. Many descriptive geometry books discuss this topic in the chapter of vector geometry or vector graphics (Earle, 1984 & Stewart, 1986). They utilize a scaled vector polygon to determine the forces in the members of a coplanar structure caused by an external load. The vector polygon, which must be constructed manually, requires vectors representing the forces be laid out end to end in continuous direction and parallel to their given positions in the space diagram. The

known vector, which represents the equilibrant (external load), is required to close the vector polygon. The force in each member can then be measured from its corresponding vector component in the vector polygon based on the same scale.

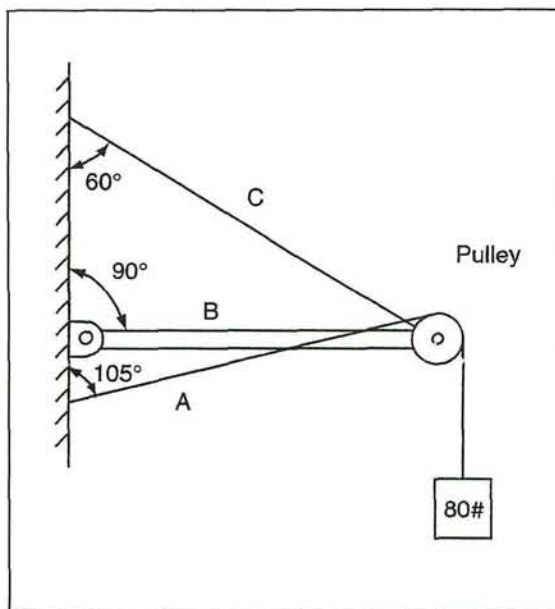
Today, CAD systems with solid modeling capability are becoming more popular in engineering design. Two researchers (Ohtshuki, et al., 1998 & Pavel, et al., 1998) investigated the potential application of 3D CAD in descriptive geometry with diversified emphasis. The former focused on the evaluation of user interface as the later focused on the direct manipulation of working drawings. Croft (1998) moved further to make positive comments on the need for descriptive geometry in a world of 3D modeling. He demonstrated how solid models could be used to extract the needed geometric information without the use of successive auxiliary views. Although every one of them addressed the possible application of 3D CAD for one topic or the other in descriptive geometry, no one discussed the application of 3D CAD for vector geometry. The pur-

pose of this study was to investigate how 3D CAD approach could be utilized in solving concurrent coplanar and non-coplanar structural problems. Both 3D CAD approach and traditional approach that requires manual construction of vector polygons are used to deal with the same set of problems, and, therefore, can be compared for their effectiveness. The 3D CAD software package utilized in this study was I-DEAS acquired from SDRC (Structural Dynamic Research Corporation). The hardware utilized was the Unix-based Sun workstation.

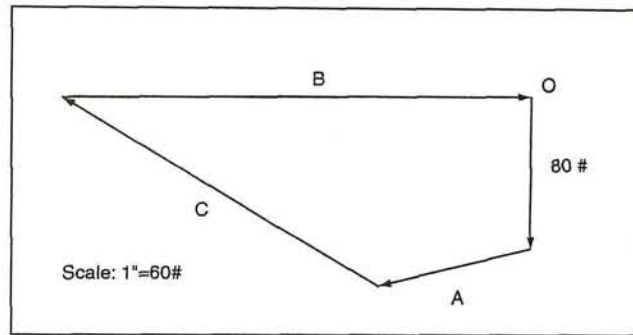
### Comparison of 3D CAD Approach to Traditional Approach

#### Concurrent Coplanar Structural Analysis

A coplanar system means that all vectors lie in the same plane. *Figure 1* depicts such a system that has coplanar forces. A pulley, which is mounted at the end of bar B, is used to support a weight of 80 pounds through cable A. If bar B is held in place by cable C, find the forces in both B and C.



*Figure 1* - Space diagram of a concurrent coplanar force system involving a pulley.



*Figure 2* - Vector polygon - traditional approach.

#### Traditional Approach

1. Draw the downward equilibrant (80 #) to scale as shown in *Figure 2*.
2. Draw vector A, which has the same length as the equilibrant vector, parallel to its direction in the space diagram (in *Figure 1*) next. This is because the loads in the cable on both sides of the pulley must be equal.
3. Draw vectors B and C parallel to their directions. To complete the vector polygon, place arrowheads of all vectors head-to-tail.
4. Find the forces in bar B and cable C by measuring vectors B and C, respectively, based on the same scale.

#### 3D CAD Approach

1. Construct a wireframe but make sure one of the edges is vertical as depicted in *Figure 3*. This can be achieved by applying the "vertical ground" command provided by the software (Lawry, 1999).
2. In order to have a fully constrained wire frame, place a linear dimension (80 for 80 pounds) and three angular dimensions (105°, 90° and 30°) using the "dimension" command. These angular dimensions are the angles between different members. For instance, 30° is the angle between cable C and bar B, the same as the complement of the angle between cable C and the wall.

3. Set the linear dimension of vector A as a reference dimension (<80>) using the "modify" command, so it would change with the linear dimension of equilibrant (80).
4. Find the forces in bar B and cable C by measuring their corresponding edges in the wireframe using either the "measurement" or "dimensioning" command.

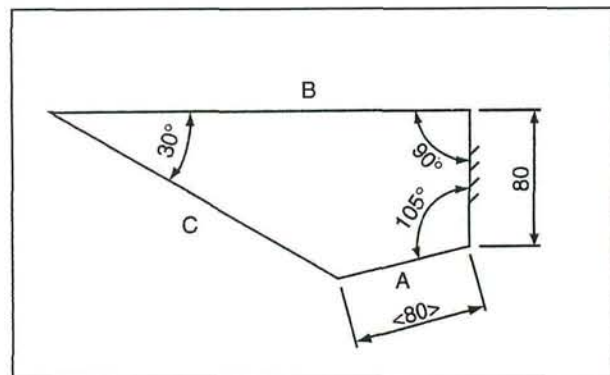


Figure 3 - Constrained wireframe - 3D CAD approach.

**Discussion**

The advantage of using constraints, including grounds and dimensions, in 3D CAD approach is that they permit the user to examine different situations quickly without the need to manually rework the vector polygon. For instance, if the weight changes (from 80 to 125 pounds) as shown in Figure 4, the force in cable A

would change automatically to match, and therefore, the forces in bar B and cable C would change accordingly. Figure 5 depicts another situation. It shows how the wireframe would adjust its shape and size automatically to accommodate the change of the angle between cable A and the wall (from 105 to 135 degrees). It is easy to see that the revision of wireframes is as swift and accurate as the use of CAD commands.

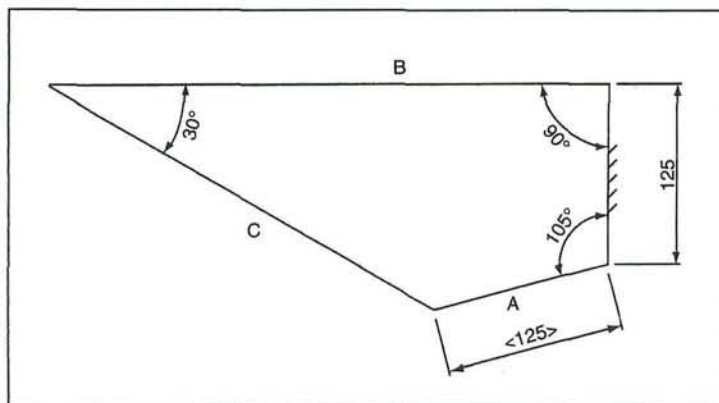


Figure 4 - Revised wireframe for a different equilibrant vector.

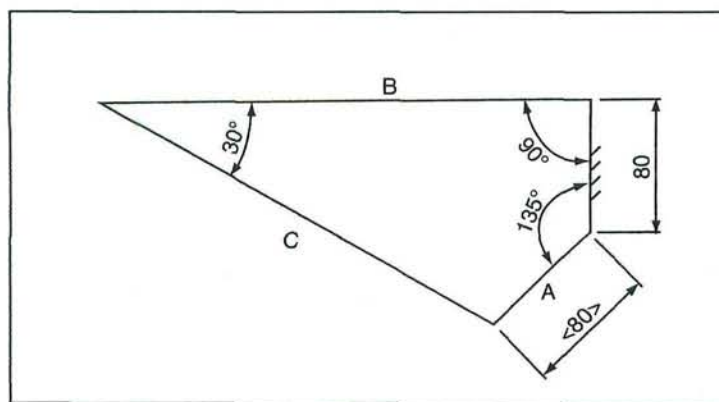


Figure 5 - Revised wireframe for a different angle.

The application of these constraints is particularly powerful while dealing with a more complex coplanar system such as a truss shown in Figure 6. Figure 7 shows the completed wireframe that represents all the forces in the truss members based on the space diagram in Figure 6. The constrained wireframe contains three vector polygons for the analysis of three different joints. These vector polygons are merged so they share at least one side. The sequence for analysis must be the same as the numerical order of the joints (in balloon) as depicted in Figure 6.

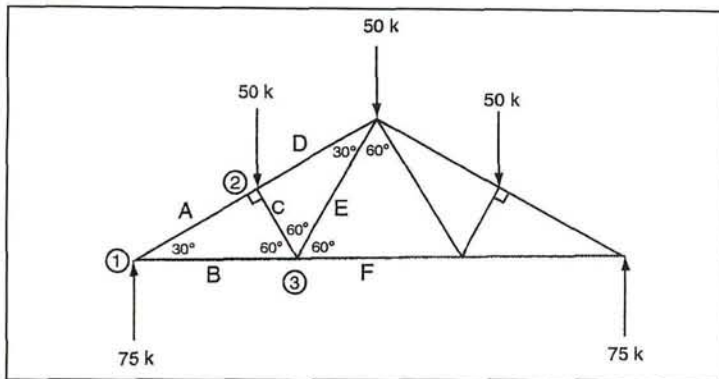


Figure 6 - Diagram of a truss with vertical loads.

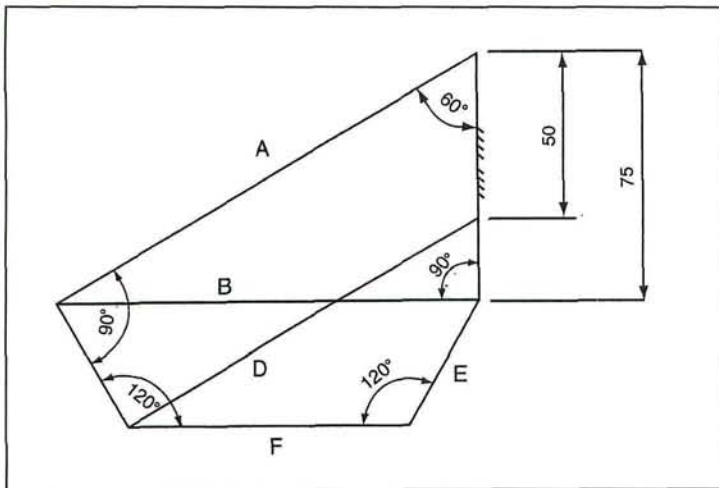


Figure 7 - Constrained wireframe for loads in truss - 3D CAD approach.

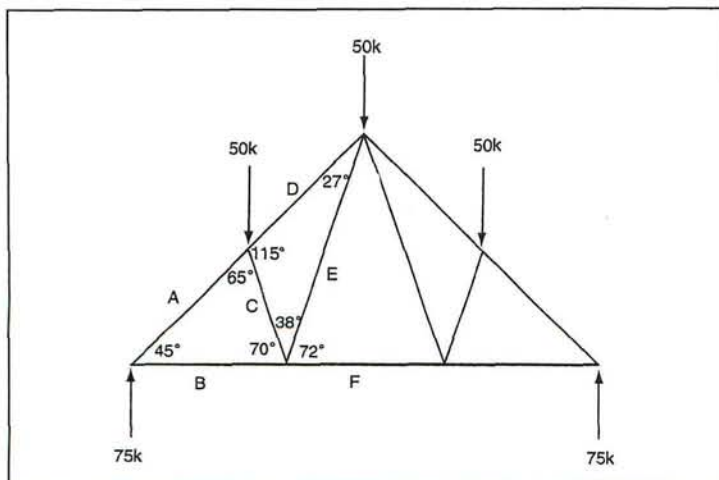


Figure 8 - Diagram of modified truss with identical loads.

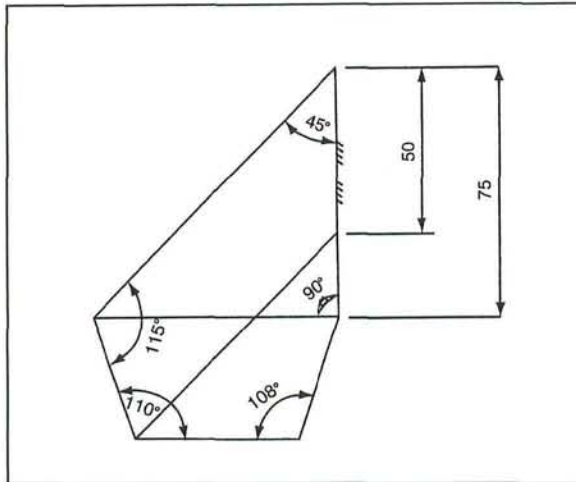
If a design alternative shown in *Figure 8* is proposed to reduce the forces in the truss members, one just has to modify the existing wireframe by changing some of the angular dimensions. *Figure 9* represents the revised wireframe based on this new design.

### Concurrent Non-Coplanar Structural Analysis: Special Case

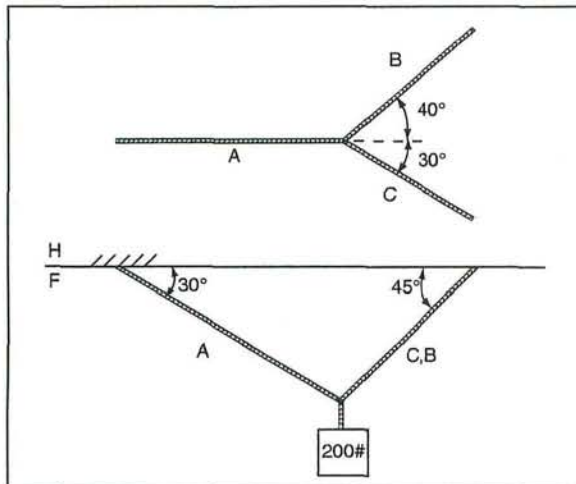
A non-coplanar system means vectors act in different planes. *Figure 10* represents such a system that has non-coplanar forces in either rigid or flexible members. In this case, three flexible cables are used to support a weight of 200 pounds. Find the tensions in all three cables if cables B and C overlap in the front view.

#### Traditional Approach

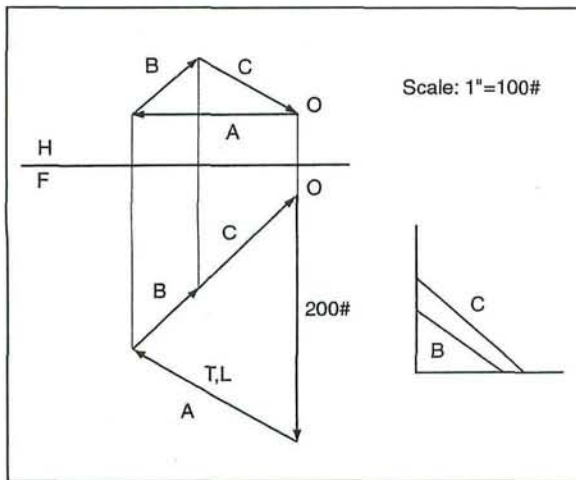
1. Draw the equilibrant vector (200 #) downward in the front view of the vector polygon as shown in *Figure 11*. The construction begins in the front view because two forces, B and C overlap here, resulting in only two unknowns in this view.
2. Draw two unknown vectors, A and "combined B and C", parallel to their directions to complete the front view of the vector polygon. Vectors B and C are inseparable, because the point of intersection of vectors B and C is unknown at this time.
3. Draw vector A in the top view of the vector polygon next. Make sure its head



**Figure 9** - Constrained wireframe for loads in modified truss.



**Figure 10** - Space diagram of a concurrent non-coplanar force system - special case.



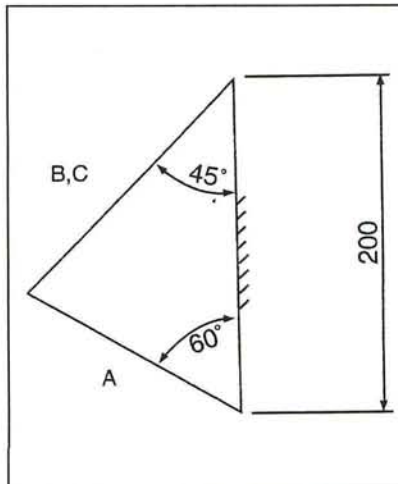
**Figure 11** - Vector polygons - traditional approach.

and tail are aligned between the top and front views. Complete the top view of the vector polygon by placing vectors B and C parallel to their directions.

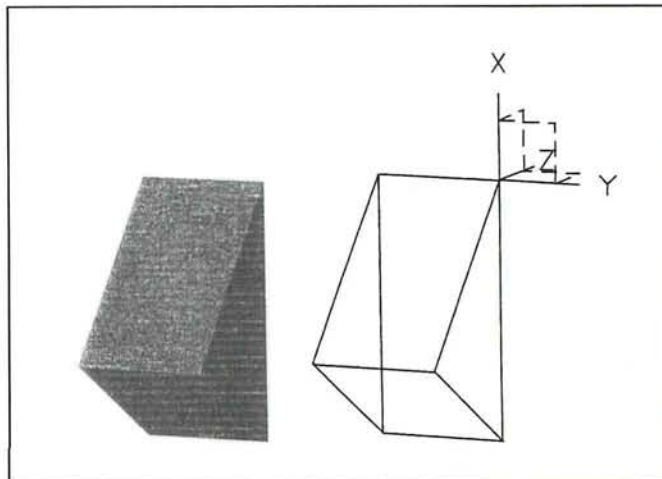
4. Project the point of intersection of vectors B and C in the top view to the front view to separate these vectors.
5. Measure the tension in cable A in the front view of the vector polygon where vector A is true length (T.L.). Construct the true-length diagram shown in *Figure 11* to determine the tensions in both cables B and C. Each of the true lengths is obtained by transferring the vertical distance between the ends of the vector to the vertical leg of the true-length diagram, and the horizontal length of the vector in the top view to the horizontal leg of the true-length diagram.

### 3D CAD Approach

1. Construct a triangular wireframe shown in *Figure 12* that represents the front view of the vector polygon. The wireframe is constrained by the equilibrant (200) with a vertical ground and the angles between the equilibrant and cables (60° and 45°).
2. Extrude the triangular wireframe into a solid. The thickness of the extrusion can be randomly selected. Repeat this step if the thickness is too small, which can be easily detected at the end of next step.
3. Attach a coordinate system to this solid at one of its upper corners as depicted in *Figure 13*. Sketch the second triangular wireframe, which represents the top view of the vector polygon, on Y-Z plane of the coordinate system. *Figure 14* represents the isometric and top views of the completed wireframe. The angles between cables (40° and 30°) in addition to the width of the solid are used to determine the shape and size of this wireframe.



**Figure 12** - Constrained wireframe - 3D CAD approach.



**Figure 13** - Solid extruded from constrained wireframe.

4. Extrude the second wireframe by intersection to obtain the final solid shown in *Figure 15*. Measure the edges as labeled to find the tensions in cables.

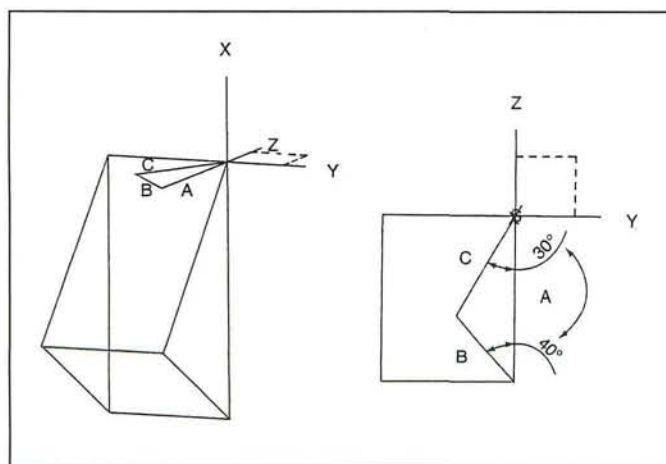
#### Discussion

When examining the design alternatives for a non-coplanar system using 3D CAD, all the changes can be carried out by applying the "history tree" command that is ideal for quick modification of a solid. The history tree displays the sequence of commands used to model a solid part. One just has to

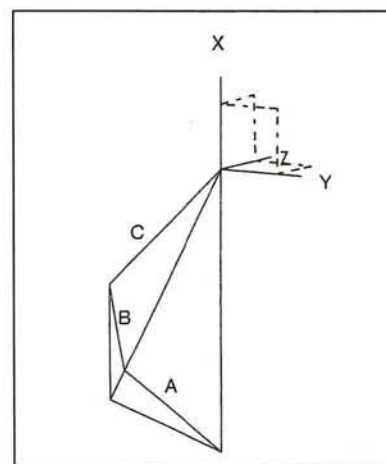
retrieve the wireframes and change their dimensions to modify the shape and size of a solid. *Figure 16* represents such a modification. The changes include (a) the equilibrant decreases from 200 pounds to 150 pounds, (b) the angle between cable A and the ceiling in the front view increases from  $30^\circ$  to  $40^\circ$ , and (c) the angle between cables B and A in the top view increases from  $40^\circ$  to  $45^\circ$ .

#### Concurrent Non-Coplanar Structural Analysis: General Case

*Figure 17* represents a problem of general



**Figure 14** - Constrained wireframe sketched on Y-Z plane of a coordinate system.



**Figure 15** - Final solid model with edges representing tensions in cables.

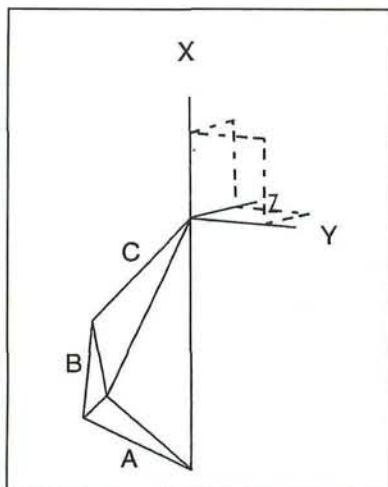


Figure 16 - Solid model with modified dimensions.

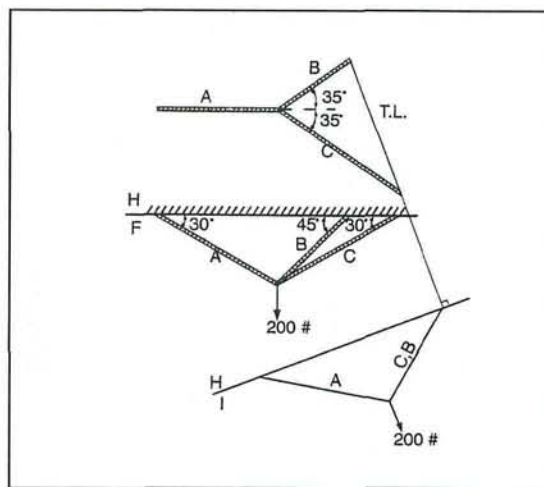


Figure 17 - Space diagram of a concurrent non-coplanar force system - general case.

case that doesn't have the overlapping of cables in either top or front view. Find the tensions in all three cables if they support a load of 200 pounds.

**Traditional Approach**

1. In order to limit the unknowns to two (instead of three), combine two of the three cables (B and C) as depicted in the primary auxiliary view of the space diagram in Figure 17. This can be achieved by obtaining the edge view of a triangle that includes any two of the three cables.
2. The procedure required next is similar to steps 1 to 4 of the traditional approach in the previous problem. The only difference is that the vector polygon must be constructed based on the top and primary auxiliary views as shown in Figure 18, instead of the top and front views of the space diagram.
3. Construct the true-length diagram to determine the tensions in all three cables.

**3D CAD Approach**

1. Construct the 3D space diagram using the "3D point" and "3D line" commands. Display the top view of the space diagram as shown at the upper left of Figure 19 using the "display of top view" command.
2. Revolve this space diagram about the X-axis of the work-plane (in dotted line) by 90 degrees, so cables B and C would appear overlapping in this new position represented on the bottom left of Figure 19.

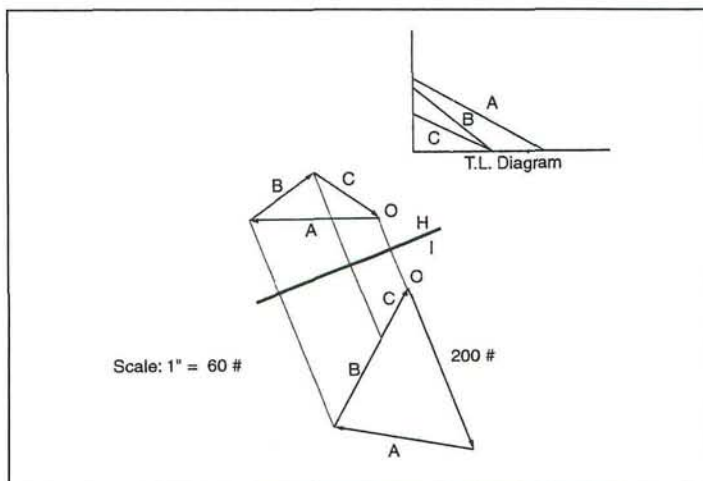
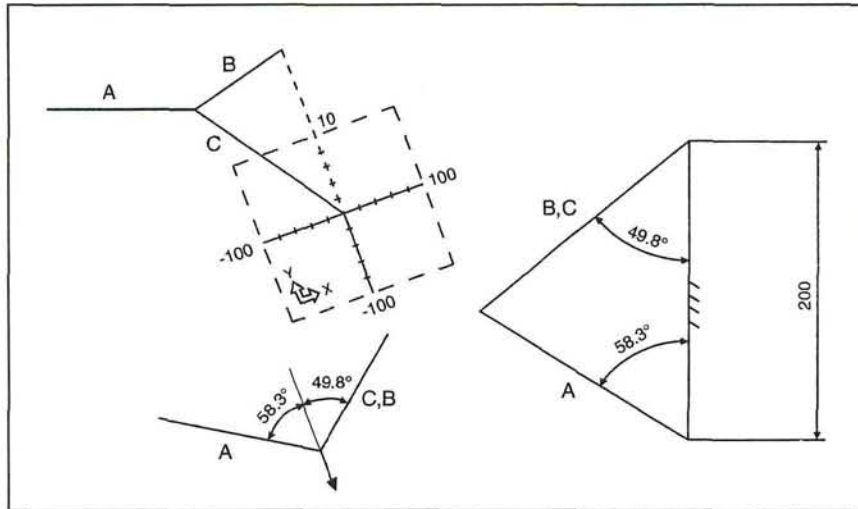
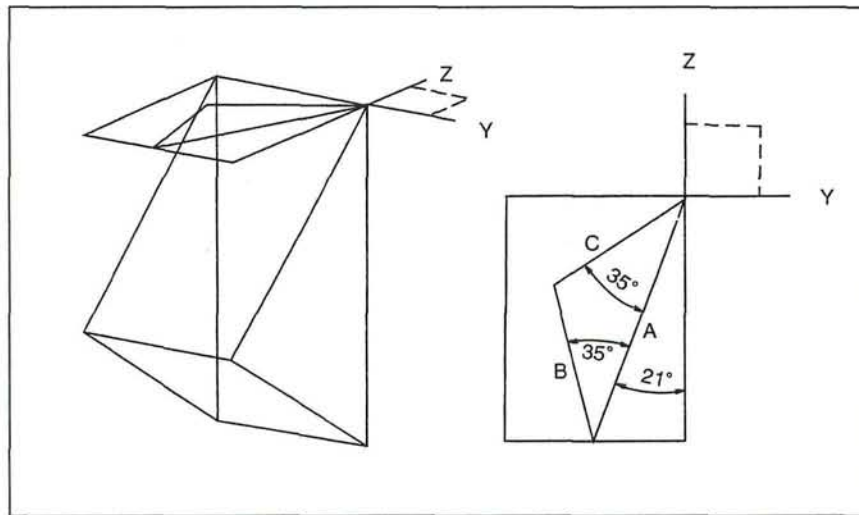


Figure 18 - Vector polygons - traditional approach.



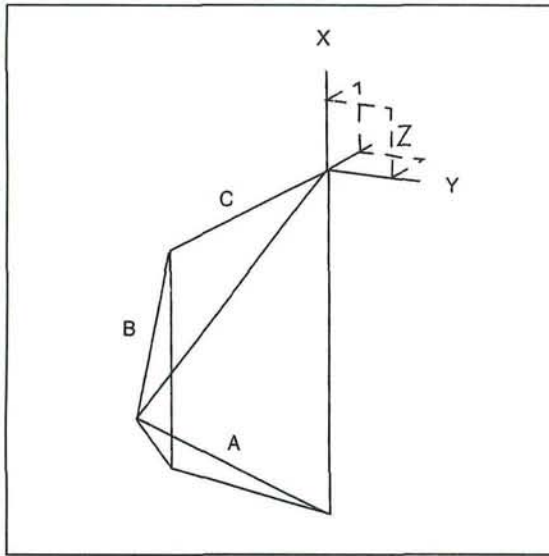
**Figure 19** - 3D CAD approach to determine angles for constrained wireframe.



**Figure 20** - Constrained wireframe sketched on Y-Z plane of a coordinate system.

3. Translate the work-plane that serves as a sketch-pad to the point of application (not shown), so the new position of the cables can be traced on the work-plane. The purpose is to determine the angles between the cables and the equilibrant in the primary auxiliary view ( $58.3^\circ$  and  $49.8^\circ$ ) as shown on the bottom left of *Figure 19*. Use these two angles in addition to the equilibrant to construct a constrained wireframe that is then extruded into a solid.
4. Construct a second wireframe using  $35^\circ$  and  $21^\circ$  angles in the top view of the space diagram ( $21^\circ$  is the angle between cable A and the folding line dividing the top view and the auxiliary view). *Figure 20* depicts how this second wireframe would be sketched on the Y-Z plane of a coordinate system. *Figure 21* represents the final solid model as a result of the extrusion by intersection. The tensions in cables can be found by measuring the edges of this solid.





**Figure 21** - Final solid model with edges representing tensions in cables.

#### Discussion

A concurrent non-coplanar structural analysis of "general case" requires a little more CAD work than that of "special case", because of the addition required in step 1. The key of 3D CAD approach for "general case" basically depends on how soon one can determine the angles between the members and the equilibrant (such as  $58.3^\circ$  and  $49.8^\circ$  from the above problem). In general, the use of 3D CAD for a general case still save a significant amount of time as compared to the manual drafting required for the traditional approach due to CAD's high performance.

#### Conclusion

The problems presented in this paper demonstrate that 3D CAD approach is much more effective in dealing with the concurrent coplanar or non-coplanar structural analysis versus traditional approach. In each of these problems, 3D CAD is less time-consuming yet more accurate, because it just requires the application of proper CAD commands instead of manual construction of scaled vector polygons. With 3D CAD approach, the analysis of other vector quantities, such

as velocity and acceleration, can also be solved quickly. Solutions can be obtained following the similar 3D CAD approach presented in this paper with minimum modifications.

Although the success of 3D CAD approach relies heavily on how effectively one can deal with various commands, it is not difficult to see that 3D CAD doesn't limit the use of geometric and spatial reasoning. As a matter of fact, the concepts and geometric rules have not changed regarding descriptive geometry solutions of vector systems. The combination of descriptive geometry with 3D CAD provides new possibilities for creative engineering design. 3D CAD can become a powerful and efficient means of learning and understanding descriptive geometry.

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